

Replicator-Mutator Dynamics and Safe Asset Status of the United States

An Evolutionary Model of the Global Safe Asset Provider
Applied to Financial Economics and Game Theory

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Abstract. This study transposes the replicator-mutator equation from evolutionary biology into the analysis of safe asset status. The model represents global safe asset demand as a population distributed across competing assets—United States Treasury securities, a diversified sovereign basket, and gold—whose fitness depends on their own market share through a network benefit net of a congestion cost, and whose holders reallocate at a rate governed by institutional disruption, the analogue of mutation. The congestion cost is the formal expression of the safe asset paradox: safety is degraded when reliance on a single asset becomes concentrated, because the price stability that constitutes safety is vulnerable to mass simultaneous liquidation through a financial system of finite intermediation capacity. The model yields four results. First, at the calibrated disruption rate a unique, locally and globally stable interior equilibrium exists in which Treasury securities command a majority of safe asset demand, while congestion bids their marginal fitness to the level of the basket. Second, erosion is self-limiting: as the incumbent’s share declines from its majority, the relief of congestion raises its relative fitness, generating a restoring force—the Minority Principle. Third, above a low threshold of disruption the equilibrium varies smoothly and reversibly with the disruption rate, without tipping point or hysteresis, and the model predicts the absence of the critical-slowness signatures that would precede a catastrophic transition. Fourth, under elevated disruption the equilibrium migrates continuously to the least-congested alternative, the diversified basket in the calibration, with gold second. The model is calibrated to the composition of the global safe asset universe—marketable safe sovereign debt and monetary gold—reproducing the current configuration. The framework integrates information-economics, congestion-game, and evolutionary approaches to safe assets within a single dynamical system, and identifies the structural conditions under which safe asset status is maintained or gradually relinquished.

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Abbreviations and Acronyms

BIS	Bank for International Settlements
CAPM	Capital Asset Pricing Model
CBO	Congressional Budget Office
CFR	Code of Federal Regulations
COFER	Currency Composition of Official Foreign Exchange Reserves (IMF database)
COVID-19	coronavirus disease 2019
CRFB	Committee for a Responsible Federal Budget
CSD	critical slowing down
EPU	Economic Policy Uncertainty (index)
ESBies	European Safe Bonds
FPU	Fiscal Policy Uncertainty (index)
FRED	Federal Reserve Economic Data (database of the Federal Reserve Bank of St. Louis)
GDP	gross domestic product
GFC	Global Financial Crisis
GPR	Geopolitical Risk (index)
IMF	International Monetary Fund
JEL	Journal of Economic Literature
NAICS	North American Industry Classification System
NBER	National Bureau of Economic Research
NQA	No Questions Asked
ODE	ordinary differential equation
OFAC	Office of Foreign Assets Control
SBU	Survey of Business Uncertainty
SDN	Specially Designated Nationals and Blocked Persons (list)
SWIFT	Society for Worldwide Interbank Financial Telecommunication
TIPS	Treasury Inflation-Protected Securities
TPU	Trade Policy Uncertainty (index)
U.S.	United States (compound-modifier form)
UST	U.S. Treasury securities

Greek Letters

γ	gamma — exponent on the network-benefit term ($0 < \gamma < 1$)
Δ	Delta — the simplex (Δ^2 : the two-dimensional simplex)
η	eta — exponent on the congestion term ($\eta > 1$)
θ	theta — generic parameter
κ	kappa — scaling constant mapping the convenience yield into σ
λ	lambda — magnitude of the flight-to-safety effect in the state-dependent extension; in Figure 12, an eigenvalue of the linearized dynamics
μ	mu — mutation (institutional-disruption) rate
σ	sigma — superiority parameter, f_1/ϕ
τ	tau — mutation-target (substitution) probability
ϕ	phi — mean fitness, $\sum_i x_i f_i$

1 Introduction

This study develops a formal dynamical model of safe asset status by transposing the replicator-mutator equation—a foundational framework in mathematical evolutionary biology—into the domain of financial economics. The model is designed to capture five features that the existing literature has identified as central to the safe asset phenomenon but has not, to the author’s knowledge, integrated within a single analytical framework: the coexistence of several safe assets at a stable interior configuration; the dual character of liquidity, which confers a network benefit at low concentration but a congestion cost at high concentration; the slow structural erosion of institutional fundamentals; the gradual and reversible character of the resulting transitions, as distinct from sudden regime change; and frequency-dependent competition among heterogeneous asset classes.

The theoretical foundations of the safe asset concept are well established. Gorton and Pennacchi (1990) demonstrated that the primary function of financial intermediaries is the production of “information-insensitive” debt—securities about which it is not profitable for any agent to become privately informed, and which can therefore circulate without engendering adverse selection. This “No Questions Asked” (NQA) property, articulated by Holmström (2015), is the condition that enables an asset to serve as money-like collateral within the wholesale financial system. Woodford (1990) and Holmström and Tirole (1998) established that government debt furnishes liquidity services that private assets cannot fully replicate, owing to the sovereign’s credibility and fiscal capacity. Bansal and Coleman (1996) showed that Treasury securities inherit a portion of the medium-of-exchange convenience of money itself, depressing the yield on Treasuries below what their credit risk alone would imply. Krishnamurthy and Vissing-Jorgensen (2012) gave these insights rigorous empirical grounding, documenting a downward-sloping demand function for the “convenience yield” on Treasury securities—the spread between the yield on safe (Aaa-rated) corporate bonds and Treasury bonds—and estimating that this spread averages approximately 73 basis points.

Three subsequent contributions completed the theoretical architecture. Brunnermeier, Merkel, and Sannikov (2024) demonstrated that safe asset status constitutes a rational bubble: the Safe Asset Tautology states that an asset is safe because it is perceived to be safe, and that the same instrument may function as a safe asset in one equilibrium and fail to do so in another. He, Krishnamurthy, and Milbradt (2019) formalized the game-theoretic structure, establishing that safe asset determination is a coordination game characterized by multiple equilibria and strategic complementarities. And the historical investigations of Schmelzing (2020) and Schmelzing, Rogoff, and Rossi (2024), which reconstruct real safe interest rates over seven centuries, established the empirical regularity that every prior safe asset provider has eventually forfeited this status.

The contribution of the present study is threefold. First, it introduces the replicator-mutator equation with network-benefit-net-of-congestion fitness as a formal model of safe asset competition. This framework, which generalizes the standard replicator dynamics of evolutionary game theory (Taylor and Jonker, 1978; Hofbauer and Sigmund, 1998) and draws on the quasispecies equation of Eigen (1971), captures the interaction between the dual liquidity effect (frequency dependence in fitness, rising then falling in own share) and institutional disruptions (modeled as mutations). Second, it establishes that erosion of safe asset status is self-limiting rather than self-reinforcing: because the dominant asset sits on the congestion-dominated branch of its fitness profile, a decline in its share raises its relative fitness and generates a restoring force (Proposition 2). This is the safe asset paradox rendered as a stabilizing principle—the Minority Principle—and, at every disruption rate above a low threshold that lies below the calibrated rate, it implies a unique

equilibrium that varies smoothly and reversibly with the disruption rate, with no error threshold, no tipping point, and no critical-slowness precursor. Third, it proves a “survival of the flattest” result: under elevated disruption the equilibrium migrates continuously to the least-congested alternative—the diversified basket—with gold second; the three-asset ordering is established by positive invariance where the inequality it rests on holds and by computation of the equilibrium branch below that rate.

The study proceeds as follows. Section 2 reviews the theoretical foundations from both financial economics and evolutionary biology. Section 3 presents the model. Section 4 derives the principal analytical results, including the self-limiting-erosion theorem (Proposition 2). Section 5 opens with a characterization of the three regimes the model generates (Definition 1) and then presents numerical simulations. Section 6 develops the self-limiting feedback loop. Section 7 establishes the absence of critical slowing down. Section 8 constructs the empirical calibration target and locates the United States on the equilibrium curve at the end of the current sample. Section 9 draws out the implications. Section 10 concludes. Appendix A contains proofs and Appendix B provides the calibration table. A companion paper (Schmid (2026)) develops the same question using a different mathematical framework—coordination games, Cournot oligopoly, and fiscal sustainability dynamics—and reaches convergent conclusions.

2 Theoretical Foundations

2.1 Information Insensitivity and the Convenience Yield

The modern theory of safe assets rests on two foundational insights. The first, due to Gorton and Pennacchi (1990) and refined by Dang, Gorton, and Holmström (2015), is that a safe asset is an asset about which it is not profitable for any agent to produce private information. This NQA property—subsequently crystallized by Holmström (2015)—is the condition enabling safe assets to circulate without engendering adverse selection, functioning as the medium of wholesale finance. The second, developed by Woodford (1990) and Holmström and Tirole (1998), is that government debt serves a distinctive and irreplaceable role as a source of private liquidity, because the sovereign’s fiscal capacity and institutional credibility endow its securities with properties that private debt cannot replicate. Krishnamurthy and Vissing-Jorgensen (2012) confirmed empirically that investors attribute a “convenience yield” to Treasury securities—a nonpecuniary return that elevates their price and depresses their yield relative to what fundamentals alone would warrant—and showed that this yield varies inversely with the outstanding supply of Treasuries relative to gross domestic product (GDP). Du, Im, and Schreger (2018) extended this analysis cross-nationally, documenting the U.S. Treasury Premium relative to other advanced sovereigns; Fleckenstein, Longstaff, and Lustig (2014) isolated the nominal Treasury-specific component through the TIPS-Treasury premium; and Fleckenstein and Longstaff (2024) provided comprehensive term-structure estimates raising concerns about the evolving nature of Treasury markets.

2.2 The Safe Asset as a Rational Bubble

The macroeconomic framework of Brunnermeier, Merkel, and Sannikov (2024) furnishes the theoretical architecture for understanding why safe asset status generates the *exorbitant privilege* and why its forfeiture withdraws that privilege. In their model, the price of a safe asset reflects not only

the present value of expected future cash flows but also the present value of future “service flows”—the value that agents derive from the capacity to retrade the asset in incomplete markets for partial self-insurance against idiosyncratic risk. This service-flow component bestows upon the safe asset a negative Capital Asset Pricing Model (CAPM) beta: its value appreciates in adverse economic states as idiosyncratic risk intensifies and the demand for self-insurance rises. The critical theoretical result is that this service-flow component constitutes a rational bubble: the Safe Asset Tautology states that an asset is safe because it is perceived to be safe, and that the same instrument, with identical underlying cash flows, may be a safe asset in one equilibrium and not in another. This self-referential character implies that safe asset status is an equilibrium concept—sustained by self-reinforcing beliefs—and that the transition away from it is nonlinear.

The same authors give the safe asset’s other defining feature, the Good Friend Analogy of Brunnermeier, Merkel, and Sannikov (2024): like a good friend, a safe asset is valuable and liquid when it is needed. Congestion qualifies this analogy rather than discarding it. The friend is reliable to each holder only on the condition that the others do not all call at once; when reliance becomes universal and holders draw on the asset together, the liquidity and price stability that made it a good friend are the first casualties of that universality. The good-friend property is realized only by the holders who do not exercise it simultaneously—the Minority Principle—and is conditional on the distribution of reliance, not a fixed attribute of the instrument.

Congestion also locates the loss of safe asset status in a mechanism different from the bubble’s. In the Safe Asset Tautology the asset is safe because it is believed safe and forfeits that status when belief is revised: the bubble pops and the system jumps between equilibria, a failure of belief. Under congestion the asset can lose the price stability that defines its safety with beliefs unchanged, through use alone, when simultaneous liquidation exceeds the system’s finite absorption capacity—the mechanism of March 2020. Failure through use is structurally distinct from failure through belief revision and does not require the equilibrium multiplicity on which the bubble rests; the same congestion that can break the asset under universal reliance is what, in place of the multiplicity of belief on which the bubble rests, yields the stable interior configuration of Section 4, unique at and above the calibrated disruption rate.

2.3 The Coordination Game

He, Krishnamurthy, and Milbradt (2019) formalized the game-theoretic structure of safe asset determination. In their model, each investor’s payoff from treating a particular asset as safe is an increasing function of the number of other investors who do likewise. The mechanism operates through liquidity: as additional investors treat an asset as safe, the depth of its secondary market increases, the bid-ask spread narrows, rollover risk diminishes, and the convenience yield rises. This generates strategic complementarities—each investor’s optimal action depends on the actions of all other investors—and the system admits multiple equilibria. The critical observation for the purposes of the present study is that these complementarities are frequency-dependent in the sense that the replicator equation from evolutionary biology is designed to capture: the fitness of a type is a function of its frequency in the population. Farhi and Maggiori (2018) extend this coordination structure to a Cournot oligopoly among sovereign issuers, introducing the Triffin dilemma: the safety premium incentivizes the hegemon to issue more debt, but overissuance eventually undermines the status that confers the premium. Gopinath and Stein (2021) identify an additional lock-in mechanism: the complementarity between a currency’s role in trade invoicing and its role as a safe store of value creates network-effect demand for dollar safe assets that delays but does not prevent the transition. The present study departs from the pure-complementarity structure of

these models in one respect that proves decisive. In the coordination game the payoff to holding an asset is everywhere increasing in the number of co-holders, which yields multiple equilibria and a tipping structure. The model developed below adds a congestion cost—a payoff that, beyond a point, decreases in the number of co-holders, because concentrated reliance degrades the safety it purports to provide. This converts the coordination game into a congestion game in the sense of the El Farol problem, with a stable interior equilibrium, and it is the source of the self-limiting dynamics established in Section 4.

2.4 Evolutionary Dynamics: From Eigen to the Replicator-Mutator Equation

The biological framework transposed into financial economics in this study possesses its own intellectual genealogy. The replicator equation, introduced by Taylor and Jonker (1978) and systematized by Hofbauer and Sigmund (1998), constitutes the dynamical equation of evolutionary game theory. In its standard form, it describes the evolution of type frequencies in a population subject to frequency-dependent selection:

$$\dot{x}_i = x_i [f_i(x) - \phi(x)], \quad (1)$$

where x_i denotes the frequency of type i , $f_i(x)$ its frequency-dependent fitness, and $\phi(x) = \sum_i x_i f_i(x)$ the population mean fitness. Types with fitness exceeding the population mean increase in frequency; types with fitness below the mean decline. The dynamics are characterized by nonlinearity inasmuch as fitness itself depends on the composition of the population.

Eigen (1971) introduced the quasispecies equation to model populations of self-replicating molecules subject to both mutation and selection. The central innovation was the mutation matrix Q , which permits imperfect copying of the master sequence. Eigen’s central result was the *error threshold*: there exists a maximum mutation rate above which the master sequence can no longer maintain its identity, and the population disperses into an undifferentiated cloud of mutants—a phenomenon termed mutational meltdown. Wilke, Wang, Ofria, Lenski, and Adami (2001) subsequently discovered the “survival of the flattest” phenomenon: at elevated mutation rates, a type possessing a lower but broader fitness peak can outcompete a type possessing a higher but narrower peak, because the former’s average fitness across its cloud of mutants exceeds that of the latter. The present study adopts the replicator-mutator scaffolding—selection coupled with mutation—but departs from Eigen’s central conclusion. By specifying fitness as non-monotonic in own share, with a congestion cost that makes the dominant type’s fitness decline at high concentration, the model replaces the error catastrophe with a stable interior equilibrium and self-limiting erosion. The quasispecies framework supplies the mathematics of mutation and selection; the economics of congestion supplies a different, non-catastrophic conclusion.

The replicator-mutator equation, which synthesizes frequency-dependent selection with mutation, assumes the form

$$\dot{x}_i = \sum_j Q_{ij} f_j(x) x_j - x_i \phi(x). \quad (2)$$

This formulation nests both the standard replicator dynamics (when Q equals the identity matrix) and the classical quasispecies equation (when fitness is frequency-independent). When both frequency dependence and mutation are present simultaneously, the resulting dynamics are richer than those generated by either component in isolation. It is this combined framework that the present study applies to the question of safe asset competition.

3 The Model

3.1 Setup and Notation

Consider a continuum of investors who must allocate their demand for safe assets among $n = 3$ competing asset classes. Let $x = (x_1, x_2, x_3)$ denote the vector of market shares, where $x_i \geq 0$ for all i and $\sum_i x_i = 1$. The state space is the two-dimensional simplex Δ^2 . The three asset classes are defined as follows.

Asset 1 (Master Sequence): U.S. Treasury securities. The incumbent safe asset provider, characterized by elevated intrinsic fitness—reflecting fiscal fundamentals and deep institutional infrastructure—and pronounced network effects arising from deep liquidity and the ubiquity of Treasuries as collateral. In the terminology of Krishnamurthy and Vissing-Jorgensen (2012), Asset 1 commands the largest convenience yield because it is the most widely held and most deeply traded safe instrument in the global financial system.

Asset 2 (Challenger): A diversified portfolio of existing national safe assets—the sovereign debt of the major non-United States issuers, held in combination rather than as a single engineered instrument. This asset achieves information insensitivity through diversification across sovereign issuers rather than through the fiscal capacity of any single sovereign. The construction is deliberately not that of a pooled-and-tranched synthetic security along the lines of the European Safe Bonds (ESBies) proposal of Brunnermeier, Langfield, Pagano, Reis, Van Nieuwerburgh, and Vayanos (2017): that proposal, and the sovereign bond-backed securities that followed it, were studied at length but not adopted, the financial-engineering route having been set aside. What exists instead is the de facto portfolio of national safe assets that reserve managers already hold, supplemented by a growing stock of joint issuance—the bonds of the European Union, on the order of one trillion euros outstanding—that constitutes a nascent single-issuer addition to the same pool rather than a replacement for it. The basket’s intrinsic safety is high, drawn from the pooled fiscal credibility of several disciplined sovereigns; its network benefit is moderate, below that of the incumbent; and its distinguishing characteristic is structural robustness to the political disruptions that afflict any individual sovereign.

Asset 3 (Robust Alternative): Commodities and gold. These assets carry high intrinsic safety—their value does not depend on the credibility of any particular institution, which accounts for gold’s historical role as a store of value over millennia—but a low network benefit, reflecting the absence of yield, the presence of storage costs, and a thin market that confers little liquidity or convenience. Their distinguishing property is near-immunity to institutional mutations.

3.2 Frequency-Dependent Fitness: Network Benefit Net of Congestion

The fitness of asset i is specified as a function of its own market share, comprising an intrinsic component, a network benefit, and a congestion cost:

$$f_i(x_i) = a_i + b_i x_i^\gamma - c_i x_i^\eta, \quad 0 < \gamma < 1 < \eta, \quad (3)$$

where $a_i > 0$ denotes intrinsic fitness—the asset’s baseline safety, reflecting freedom from default and institutional risk, fiscal fundamentals, and the rule of law; $b_i \geq 0$ denotes the strength of the network benefit (the liquidity and convenience premium that accrues to a widely held asset); and $c_i \geq 0$ denotes the strength of the congestion cost: the price response that mass simultaneous liquidation would provoke, and hence the rate at which the asset’s price stability—the substance

of its safety—degrades as reliance on it concentrates. The coefficient is an inverse measure of absorption capacity, which rises with the depth of the asset’s market and with the degree to which reliance is spread across decorrelated submarkets rather than concentrated in one. The exponent $\gamma < 1$ governs the concavity of the network benefit, capturing diminishing marginal returns to liquidity depth: the marginal convenience value of an additional unit of the asset declines as its share rises. This is the relationship documented by Krishnamurthy and Vissing-Jorgensen (2012), whose downward-sloping demand curve for the convenience services of Treasury securities implies that the marginal holder values convenience less as the outstanding stock expands. The exponent $\eta > 1$ governs the convexity of the congestion cost, which rises more than proportionally with share.

The congestion term is the formal expression of the safe asset paradox. Safety is a property realized only by holders who are not all relying on the asset at once: when a single instrument becomes the near-universal repository of safe value, the price stability that constituted its safety becomes vulnerable to mass simultaneous liquidation through a financial system of finite intermediation capacity. The asset’s safety is, in this sense, degraded by its own ubiquity—the Minority Principle, which the companion work develops through the congestion structure of the El Farol problem. The empirical support for this term is not the convenience-yield literature, which concerns the scarcity pricing of an undiminished service (Caballero, Farhi, and Gourinchas 2017), but the episodes in which the safe asset failed through use—most directly the March 2020 “dash for cash,” when Treasuries sold off as holders sought to liquidate simultaneously and the price stability that defined the asset broke under the weight of universal reliance, restored only by the central bank’s balance sheet.

The absorption-capacity reading orders the three assets. Gold trades in the thinnest market and cannot be diversified across issuers, so a given mass liquidation moves its price the most: it is the most congestion-prone asset, with the largest coefficient c_3 . The diversified basket sits at the opposite pole, since reliance on it is spread across several decorrelated national markets, each absorbing a fraction of any liquidation, so its aggregate price response is the smallest and c_2 the least. U.S. Treasuries fall between: theirs is the deepest single market, but reliance on them is highly correlated—they are the common collateral of the financial system—and concentrated in one dealer sector of finite balance-sheet capacity, so their stress-time price response is large despite their depth, as March 2020 showed. The ordering is thus $c_3 > c_1 > c_2$: gold most congested, the basket least, the incumbent between. The incumbent’s failure through congestion in equilibrium follows not from the largest coefficient but from the largest share, since the realized cost $c_i x_i^\eta$ is governed jointly by the coefficient and the concentration of reliance.

The interaction of the two share-dependent terms generates a fitness profile that rises at low share, where the network benefit dominates, and falls at high share, where congestion dominates. The interior maximum of f_i thus represents the share at which the asset’s safety is most fully realized; beyond it, additional reliance is self-defeating. It is this non-monotonicity—fitness decreasing in own share at high concentration—that admits a stable interior equilibrium in which the three assets coexist, a configuration that a purely network-driven (monotonically increasing) fitness cannot support.

Figure 1 displays the fitness functions for the three asset classes. U.S. Treasuries (depicted in dark blue) carry the strongest network benefit (b_1); gold (brown) carries the largest congestion coefficient (c_3) and the diversified basket (amber) the smallest (c_2), so the gold profile is the most attenuated by congestion and the basket profile the flattest. The incumbent’s profile is the tallest, lifted by its network benefit, and is congestion-dominated at its calibrated share through the concentration of reliance rather than through the size of its coefficient. At the calibrated equilibrium,

discussed in Section 5.2, U.S. Treasuries hold a share of approximately 62 percent; their fitness exceeds the population mean, while their marginal fitness has been bid down by congestion to slightly below that of the basket—a configuration examined in Section 4.1.

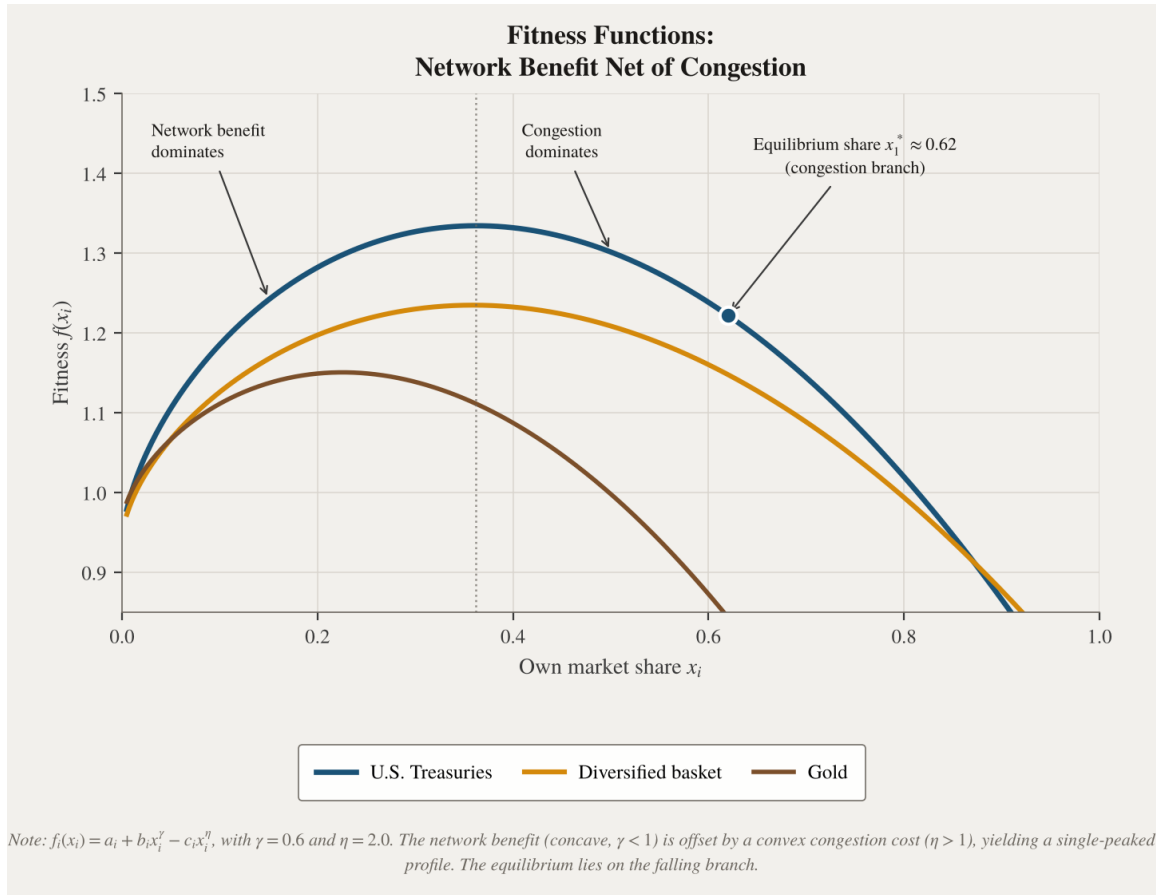


Figure 1. Frequency-Dependent Fitness Functions: Network Benefit Net of Congestion

3.3 The Mutation Matrix

Let Q denote a 3×3 column-stochastic matrix, where Q_{ij} represents the probability that a holder of asset j transitions to asset i in a given period. The diagonal elements $Q_{jj} = 1 - \mu_j$ represent faithful replication—that is, the probability that an investor continues to hold the same asset—where μ_j denotes the total mutation rate for asset j . The off-diagonal elements $Q_{ij} = \mu_j \cdot \tau_{ij}$ specify the direction of mutations, where τ_{ij} represents the probability that a mutating holder of asset j transitions specifically to asset i , subject to the constraint $\sum_{i \neq j} \tau_{ij} = 1$ for each j .

The mutation rate μ_1 pertaining to U.S. Treasuries constitutes the central parameter of the model. In the biological framework, it represents the per-generation probability of a copying error in the master sequence. In the financial transposition, it captures the frequency of institutional disruptions that degrade the NQA property of the asset. Such disruptions encompass debt-ceiling confrontations, partisan fiscal brinkmanship, encroachments on central bank independence, the weaponization of the payments infrastructure through financial sanctions, and policy reversals that augment uncertainty regarding the legal and fiscal framework governing the public debt. Each such

event constitutes a “mutation” in the precise sense of Eigen (1971): an imperfect replication of the institutional conditions that sustain the asset’s safe status from one period to the next.

3.4 The Replicator-Mutator Ordinary Differential Equation

The dynamics of the system are governed by the Ordinary Differential Equation (ODE) of (2),

$$\dot{x}_i = \sum_j Q_{ij} f_j(x) x_j - x_i \phi(x), \quad i = 1, 2, 3,$$

where $\phi(x) = \sum_i x_i f_i(x)$ denotes the population mean fitness. The first term on the right-hand side represents the inflow of market share to asset i from all assets j (including asset i itself), weighted by the fitness of each source asset and the probability of transition. The second term ensures that the state vector remains confined to the simplex, so that the constraint $\sum_i x_i = 1$ is preserved under the dynamics. This formulation nests the standard replicator dynamics as a special case (when $Q = I$) and the classical quasispecies equation as another (when the fitness functions are frequency-independent).

4 Analytical Results

4.1 Existence of a Congestion-Stabilized Dominant Equilibrium

Proposition 1 (Existence and Stability of a U.S.-Dominant Equilibrium). *For mutation rates μ_1 below the rate at which the incumbent cedes the majority, the replicator-mutator system with network-benefit-net-of-congestion fitness of the form (3), at the calibrated parameters, possesses an interior equilibrium x^* that is locally asymptotically stable and satisfies $x_1^* > 1/2$. On this branch three properties hold. First, U.S. Treasuries command a majority of global safe asset demand, and for μ_1 above about 0.010, a range that contains the calibrated rate $\mu_1 = 0.02$, their fitness exceeds the population mean, so the superiority parameter*

$$\sigma(x^*) \equiv \frac{f_1(x_1^*)}{\phi(x^*)}$$

exceeds unity. Second, for μ_1 up to about 0.021 (the calibrated rate lies in this range), the incumbent’s marginal fitness is bid below that of the diversified basket, $f_2(x_2^) > f_1(x_1^*)$: congestion has competed the safety premium on the dominant asset down to, and slightly below, that of the basket. Third, the equilibrium lies on the congestion-dominated branch of the incumbent’s fitness profile, $f_1'(x_1^*) < 0$, and it is this negative own-share dependence that furnishes the stability the equilibrium possesses—a property a monotonically increasing (purely network-driven) fitness cannot supply.*

Proof sketch. An interior rest point satisfies the stationarity conditions $\dot{x}_i = 0$ for $i = 1, 2, 3$. Because each fitness function (3) is single-peaked in own share—rising while the network benefit dominates and falling once congestion dominates—the dominant asset’s stationary share lies on the downward-sloping branch, where $f_1'(x_1^*) < 0$. The negative own-share dependence renders the diagonal of the Jacobian of the reduced two-dimensional system strictly negative, and for the calibrated parameters both eigenvalues have strictly negative real parts, so the equilibrium is locally

asymptotically stable; at the calibrated rates, numerical integration from arbitrary initial conditions on the simplex confirms that it is globally attracting. The incumbent's stationarity condition balances the excess of its fitness over the mean against the net mutation flow, $(f_1 - \phi) x_1^* = \mu_1 f_1 x_1^* - \sum_{j \neq 1} \mu_j \tau_{1j} f_j x_j^*$: the superiority exceeds unity exactly where the mutation outflow from the incumbent exceeds the mutation inflow to it, which at the calibrated parameters holds for μ_1 above about 0.010. Computed along the branch, $f_2(x_2^*) > f_1(x_1^*)$ holds for μ_1 up to about 0.021. The full proof is provided in Appendix A. $\square$

4.2 Self-Limiting Erosion and the Restoring Force

Proposition 2 (Self-Limiting Erosion). *In a neighborhood of the U.S.-dominant equilibrium, the superiority parameter is increasing in the mutation rate,*

$$\frac{\partial \sigma(x^*(\mu_1))}{\partial \mu_1} > 0, \quad (4)$$

or equivalently $\partial \sigma / \partial x_1^ < 0$ along the equilibrium manifold: as the incumbent's market share declines, its fitness relative to the population mean rises. A disruption that erodes the incumbent's share therefore raises the incumbent's relative fitness and generates a restoring force toward the equilibrium. The feedback is self-limiting, not self-reinforcing.*

Proof sketch. The incumbent's fitness $f_1(x_1) = a_1 + b_1 x_1^\gamma - c_1 x_1^\eta$ is single-peaked, and the dominant equilibrium lies on its downward-sloping branch, where $f'_1(x_1^*) < 0$. A reduction in x_1^* therefore raises $f_1(x_1^*)$: the relief of congestion ($-c_1 x_1^\eta$ falling in magnitude) outweighs the loss of network benefit ($b_1 x_1^\gamma$ declining) on this branch. Because the displaced share migrates to assets whose fitness contributions to the population mean ϕ are not larger by more than an offsetting amount, the ratio $\sigma = f_1/\phi$ rises. The mechanism is the Minority Principle made dynamical: the safety of the dominant asset is most degraded when reliance upon it is most concentrated, so a withdrawal of reliance restores that safety, opposing the disruption that induced it. The full argument, including the sign of $\partial \sigma / \partial \mu_1$ under the calibrated parameters, is given in Appendix A. $\square$

This result reverses the central dynamic of the classical quasispecies model and of the network-effect formulation. In the Eigen framework, a master sequence whose share is eroded loses fitness, so erosion compounds: the system either maintains the master sequence or collapses through an error catastrophe. Under network-benefit-net-of-congestion fitness the opposite holds near the equilibrium. Erosion is self-arresting because the property that confers fitness—safety—is degraded by concentration and restored by its relief. There is, in consequence, no error threshold of the Eigen type and no tipping point along the dominant branch; the system does not stand a fixed distance from a catastrophe that erosion brings nearer. This is the formal counterpart of the restoring force the empirical literature has invoked, without formalizing, in accounting for the episodes of partial recovery in the dollar's reserve share. The restoring force established here is a property of the disruption channel: a structural increase in the incumbent's congestion cost, as distinct from a disruption that erodes its share, is not opposed by the same mechanism, a channel-specificity established in Section 4.5.

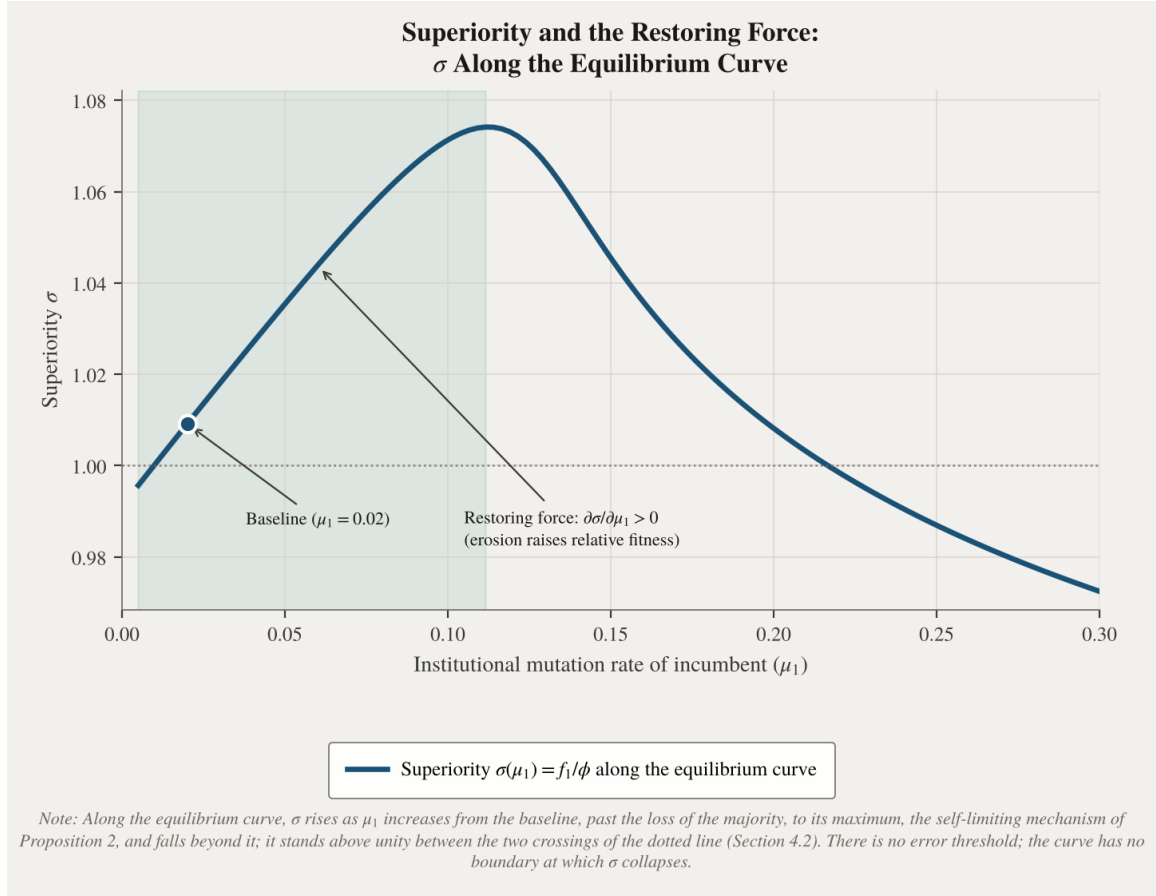


Figure 2. Superiority and the Restoring Force: σ as a Function of the Mutation Rate

Figure 2 displays the superiority parameter $\sigma(x^*(\mu_1))$ as a function of the mutation rate. Beginning from the calibrated equilibrium, σ rises as the mutation rate increases and the incumbent's share declines, past the loss of the majority at $\mu_1 \approx 0.055$ to its maximum near $\mu_1 = 0.112$, and declines at higher disruption; it stands above unity for μ_1 between about 0.010 and 0.217 (Appendix A). The ascending segment is the domain of the restoring force established in Proposition 2: in this range, erosion of the incumbent's share strengthens, rather than weakens, its relative fitness.

4.3 Comparative Statics and the Smooth Transition

Proposition 3 (Comparative Statics). *At the U.S.-dominant equilibrium x^* , the following comparative statics obtain:*

- (i) $\partial x_1^*/\partial \mu_1 < 0$, $\partial x_2^*/\partial \mu_1 > 0$, and $\partial x_3^*/\partial \mu_1 > 0$;
- (ii) $\partial x_1^*/\partial a_1 > 0$ and $\partial x_1^*/\partial b_1 > 0$;
- (iii) $\partial x_1^*/\partial c_1 < 0$;
- (iv) $\partial \sigma/\partial \mu_1 > 0$ in a neighborhood of the equilibrium (Proposition 2);

(v) for μ_1 above the low-disruption threshold of Section 5.5 ($\mu_1 \approx 0.0074$), the map $\mu_1 \mapsto x^*(\mu_1)$ is continuous and single-valued: the equilibrium varies smoothly with the mutation rate, without saddle-node bifurcation or hysteresis.

In qualitative terms: a rise in institutional disruption erodes the incumbent's equilibrium share and disperses it to both alternatives (i); the share is increasing in the incumbent's intrinsic fitness and network benefit (ii) and decreasing in its congestion cost (iii); and the superiority parameter rises rather than falls as disruption increases, the restoring force of Proposition 2 (iv). The final property (v) is the structural signature of the congestion mechanism: along the dominant branch the Jacobian remains non-singular (Appendix A), so above the low-disruption threshold the equilibrium manifold is a smooth curve in μ_1 , the incumbent's share decreasing along it, rather than a branch that loses existence at a critical value. There is no point on it at which the dominant equilibrium is annihilated and the system jumps discontinuously to an alternative configuration.

Corollary 1 (No Catastrophe; Bounded and Reversible Erosion). *For every μ_1 above about 0.0074, a range that contains the calibrated rate, the U.S.-dominant equilibrium is unique and varies continuously and reversibly with μ_1 . Over that range there is no error threshold of the Eigen type and no tipping point: erosion induced by an increase in μ_1 is bounded, and a subsequent decline in μ_1 restores the prior share along the same equilibrium curve. Over that range the system exhibits no hysteresis. Below that rate a second stable equilibrium coexists with the dominant one (Section 5.5).*

This corollary reconciles the model with an empirical pattern that a self-reinforcing mechanism cannot accommodate: the episodes of partial recovery in the dollar's reserve share, in which a period of decline was followed by a renewed rise rather than by continued erosion. Under network-benefit-net-of-congestion fitness such reversals are the expected response to a recession of disruption, because the restoring force of Proposition 2 operates symmetrically. The classical quasispecies model differs: there, the master sequence is either maintained or lost through an irreversible catastrophe; here, the dominant share drifts reversibly with the disruption rate.

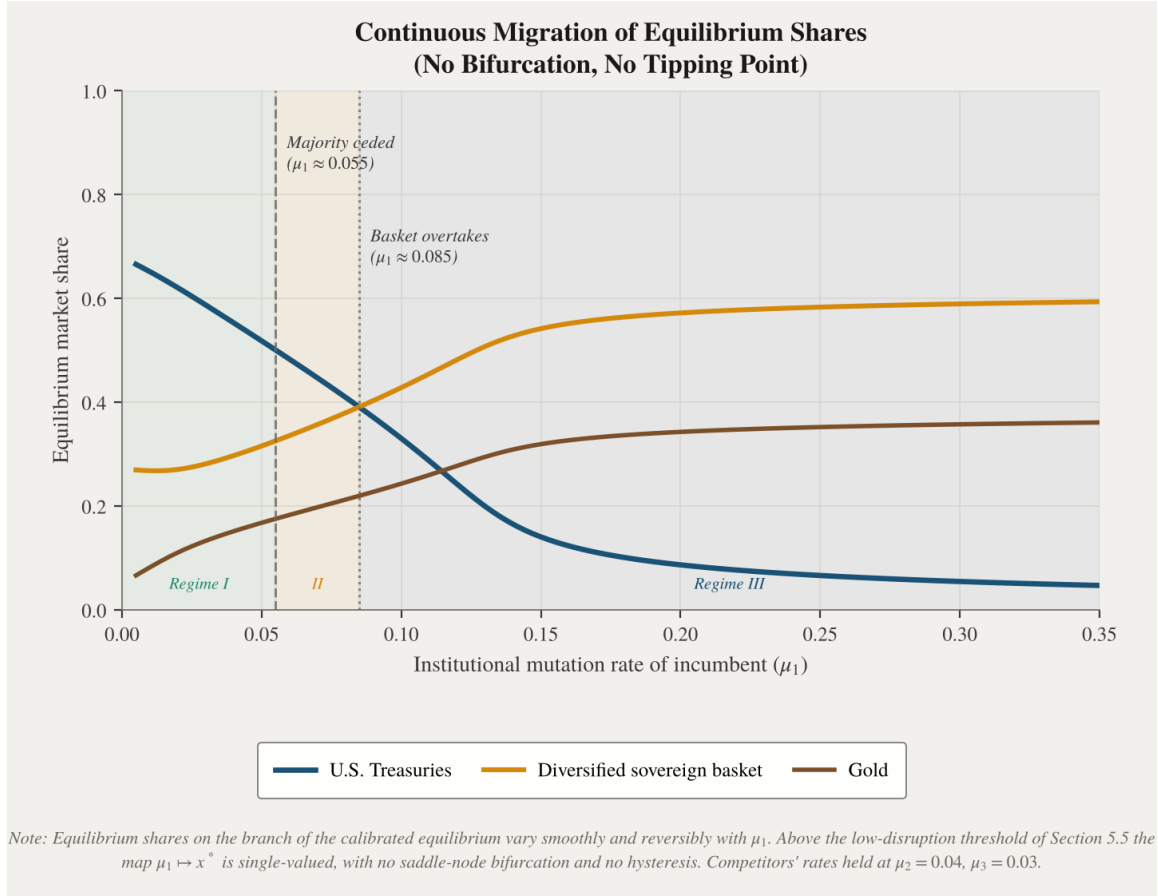


Figure 3. Equilibrium Market Shares as a Continuous Function of the Incumbent's Mutation Rate

Figure 3 traces the equilibrium market shares of U.S. Treasuries (dark blue), the diversified basket (amber), and gold (brown) as functions of the mutation rate μ_1 , with all other parameters held at their baseline values. The incumbent's share declines continuously from its calibrated majority, ceding share to both alternatives, while the basket—the least congested asset—rises to assume the majority at high disruption. Along this branch the transition is smooth throughout: there is no critical value at which the curves break, and the same curve is traced whether μ_1 is increased or decreased, reflecting the absence of hysteresis established in Corollary 1.

4.4 Survival of the Flattest

Proposition 4 (Survival of the Flattest). *As the incumbent's mutation rate μ_1 rises, its equilibrium share x_1^* declines continuously and the displaced demand accrues to the alternatives. The diversified basket emerges as the plurality holder: its share x_2^* rises monotonically in μ_1 from the calibrated rate onward and overtakes the incumbent's, so that for μ_1 above a transition rate the ordering $x_2^* > \max(x_1^*, x_3^*)$ obtains. The basket prevails over gold among the alternatives on three counts: it is the least congested asset and hence the flattest profile, able to absorb reliance with the smallest price response; it carries the larger network benefit ($b_2 > b_3$); and it receives the larger share of substitution flows out of the incumbent ($\tau_{21} > \tau_{31}$).*

Gold, the most congested asset, is held to a secondary share, its thin market limiting the reliance it can absorb. The successor configuration is thus the diversified basket, approached gradually and not through a discontinuous transition.

The ordering $\tau_{21} > \tau_{31}$ is a property of the holder base rather than a free primitive. The substitution shares are averages over holders, and the destination of displaced demand depends on the motive for displacement. A holder fleeing a loss of liquidity moves to the deepest alternative market—the sovereign basket, not gold—so that $\tau_{21} > \tau_{31}$, in keeping with $b_2 > b_3$. A holder fleeing the seizure risk exposed by the weaponization of reserves gains nothing from the basket, whose non-incumbent sovereign claims are reachable by the same coalition, and moves to unencumbered gold, so that $\tau_{31} > \tau_{21}$. The aggregate ordering is the weight of the two motives in the population: the basket’s larger accumulated stock (Section 8) places that weight at $\tau_{21} > \tau_{31}$, while episodes of gold accumulation by sanction-exposed sovereigns enter as a shift in the weight, not a violation of the ordering.

The label *survival of the flattest* carries over from the quasispecies literature, and under the congestion ordering it holds literally. There, the broad, flat fitness peak outlasts the tall, narrow one under high mutation because it is robust to dispersal. Here, as disruption erodes the incumbent’s majority, the displaced reliance flows to the flattest of the remaining profiles. The basket, least attenuated by congestion because reliance on it is diversified across decorrelated markets, absorbs additional reliance with the smallest degradation of its safety; gold, the most congested and hence the sharpest profile, congests too quickly to take more than the second share. The mechanism is congestion-relief and substitution rather than mutational dispersal, but the selection logic is the same: peak height does not secure survival under perturbation, the asset that can sustain the largest reliance without self-defeating is the flattest, and it is the flattest that prevails. Concentration gives way to diversification, as in the quasispecies case, by a structurally distinct route.

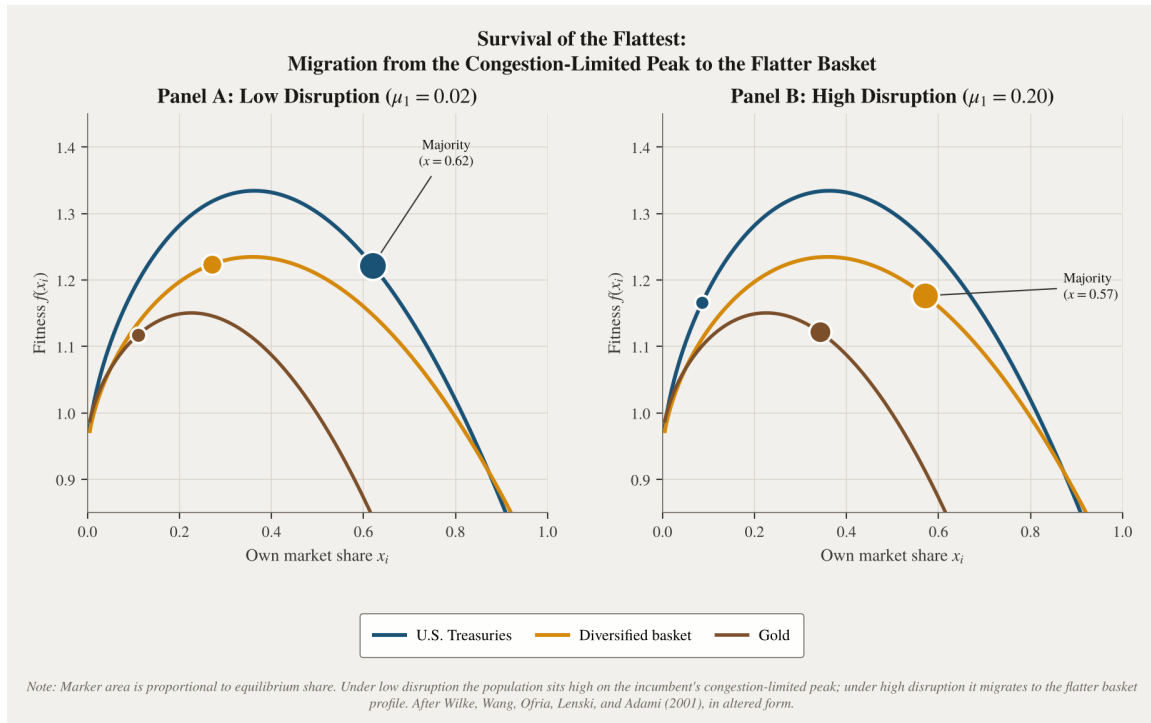


Figure 4. Survival of the Flattest — Continuous Migration to the Diversified Basket as Disruption Rises

Figure 4 renders the result. Each panel depicts the three single-peaked fitness profiles—U.S. Treasuries (dark blue), the diversified basket (amber), and gold (brown)—together with the location of the population on the share simplex. The incumbent’s profile is the tallest, lifted by its network benefit, and is congestion-dominated at its high calibrated share; gold’s profile is the most attenuated, peaking early and falling steeply; the basket’s is the flattest. Panel A shows the low-disruption configuration, in which the population sits at the incumbent’s congestion-limited peak and Treasuries hold the majority. Panel B shows the high-disruption configuration, in which the population has migrated toward the flatter basket profile, which now holds the majority. The migration between the two is smooth, traced by the equilibrium curve of Figure 3. The result furnishes the theoretical foundation for the prediction that the successor to U.S. Treasury dominance, under elevated institutional disruption, is the diversified, least-congested instrument among the alternatives—reached by gradual reallocation rather than by collapse.

4.5 Asymmetry of the Two Erosion Channels

The restoring force of Proposition 2 was derived for the disruption channel, in which a rising mutation rate μ_1 erodes the incumbent’s share. The incumbent’s share is also decreasing in its own congestion cost c_1 (Proposition 3(iii)), so a structural increase in c_1 —a permanent reduction in the absorption capacity supplied by the holder base and the intermediation channels, as opposed to a transient disruption—is a second route by which the share erodes. The two routes are not equivalent in their effect on relative fitness.

Proposition 5 (Channel-Specificity of the Restoring Force). *The restoring force of Proposition 2 is a property of the disruption channel and does not extend to a structural increase in the incumbent’s congestion cost. Along the equilibrium manifold,*

$$\frac{\partial \sigma}{\partial \mu_1} > 0 \quad (\text{Proposition 2}), \quad \frac{\partial \sigma}{\partial c_1} < 0.$$

At a common incumbent share the two channels place the superiority on opposite sides of its baseline value: a share eroded by disruption carries a superiority above baseline, while the same share reached by raising c_1 carries a superiority below it. Under the congestion channel σ declines through unity as c_1 rises—the value at which the incumbent’s fitness falls to the population mean.

Proof sketch. Differentiating $\sigma = f_1(x_1^*)/\phi$ along the c_1 manifold,

$$\frac{\partial \sigma}{\partial c_1} = \frac{1}{\phi} \left[\frac{\partial f_1}{\partial c_1} + f_1'(x_1^*) \frac{\partial x_1^*}{\partial c_1} \right] - \frac{f_1}{\phi^2} \frac{\partial \phi}{\partial c_1}, \quad \frac{\partial f_1}{\partial c_1} = -(x_1^*)^\eta < 0.$$

The direct term lowers the incumbent’s fitness at fixed share. The indirect term is positive on the falling branch, where $f_1'(x_1^*) < 0$ and $\partial x_1^*/\partial c_1 < 0$ (Proposition 3(iii)): the share that the coefficient increase sheds relieves congestion and partially offsets the direct penalty. At the calibrated parameters the direct effect dominates and the net is negative, $\partial \sigma/\partial c_1 = -0.022$, against $\partial \sigma/\partial \mu_1 = +0.890$ on the disruption channel. The sign is the reverse of Proposition 2; the full computation is given in Appendix A. $\square$

Figure 5 traces the two channels on a common axis, the incumbent’s equilibrium share. Both lower the share from the calibrated 0.62; along the disruption channel the superiority rises—the

self-limiting restoring force of Proposition 2—while along the congestion channel it falls, crossing unity near $x_1^* \approx 0.48$ (at $c_1 \approx 1.75$, against the calibrated $c_1 = 1.30$). Under the congestion channel the incumbent cedes the majority at $c_1 \approx 1.66$, a factor of 1.28 above the calibrated value, and the basket overtakes it at $c_1 \approx 2.09$, a factor of 1.61 above. At the calibrated disruption rate the equilibrium remains unique at every c_1 from 0.05 to 6: the congestion channel is a smooth slide along a migration curve, not a loss of stability, so the no-tipping-point and reversibility results of Section 4.3 are undisturbed.

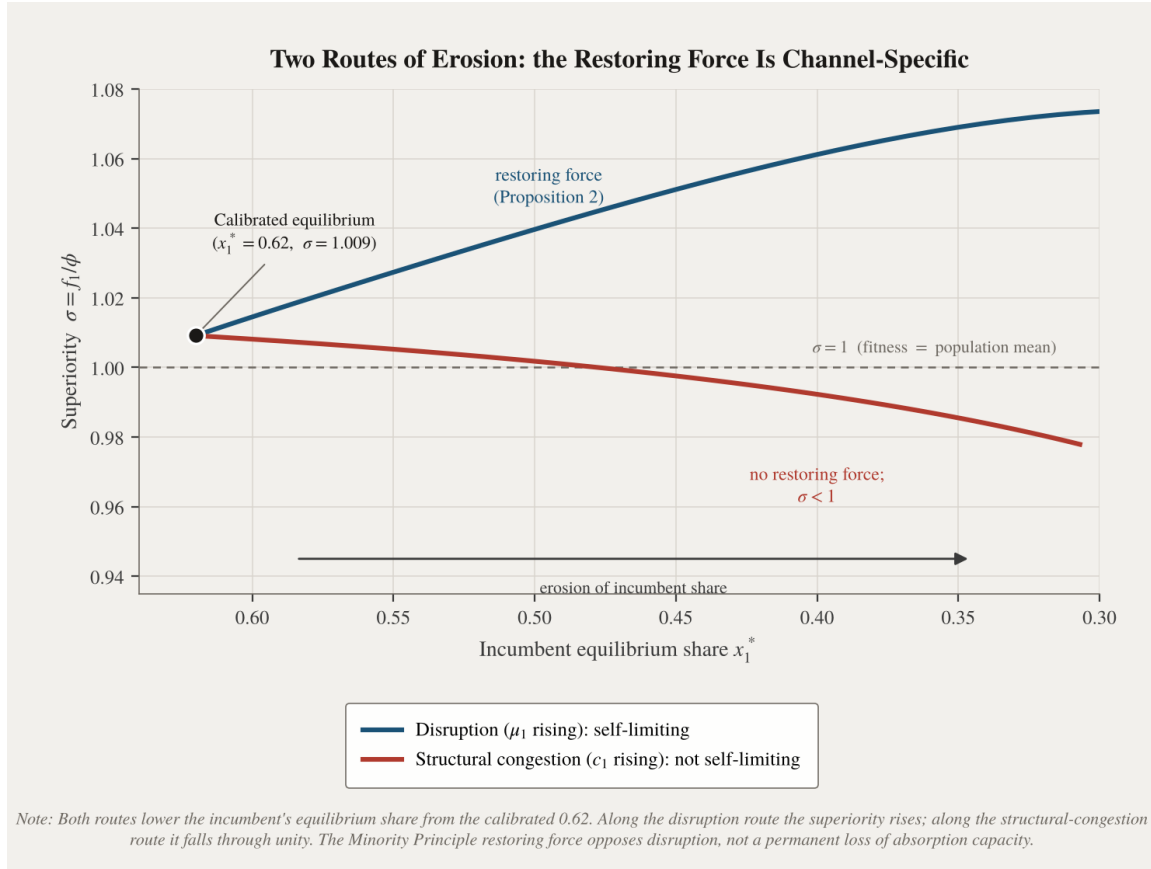


Figure 5. Two Routes of Erosion: the Restoring Force Is Channel-Specific

The distinction has an economic reading. A disruption is transient and, by Proposition 2, self-correcting: the share lost as disruption rises is recovered as it recedes, along a reversible curve. A structural increase in c_1 is permanent and is not offset by the restoring force, because the coefficient governing the price response to mass liquidation has itself risen. The two are the analytical counterparts of distinct sources of erosion—episodic institutional disruption on the one hand, and a secular concentration of reliance or thinning of intermediation capacity on the other. The lever that acts on the second, developed in Section 9.4, is the structural diversity of the holder base that lowers c_1 ; it operates on the one channel of erosion that the restoring force leaves unopposed.

5 Numerical Analysis and Simulation

5.1 Regime Structure of the Dynamic Trajectory

Before presenting the numerical simulations, this subsection formally defines the three regimes into which the equilibrium curve is partitioned by the analytical results of Section 4. These labels recur throughout the remainder of the paper.

Definition 1 (Regime Structure). Let $x^*(\mu_1)$ denote the branch of equilibria that continues the congestion-stabilized dominant equilibrium of Proposition 1, let $\underline{\mu}_1$ denote the transition rate at which the incumbent cedes the majority, $x_1^*(\underline{\mu}_1) = 1/2$, and let $\bar{\mu}_1$ denote the rate at which the basket overtakes it, $x_2^*(\bar{\mu}_1) = x_1^*(\bar{\mu}_1)$. The equilibrium curve is partitioned into three regimes:

- **Regime I (Stable majority):** $\mu_1 < \underline{\mu}_1$. The system rests near the dominant equilibrium with $x_1^* > 1/2$; U.S. Treasuries hold the majority, and the superiority parameter $\sigma(x^*)$ exceeds unity for μ_1 above about 0.010 (Proposition 1); below $\mu_1 \approx 0.0074$ a second stable equilibrium coexists with it (Section 5.5).
- **Regime II (Erosion of the majority):** $\underline{\mu}_1 \leq \mu_1 < \bar{\mu}_1$. The equilibrium share x_1^* declines continuously through $1/2$, dispersing to both alternatives; by Proposition 2 the superiority σ is rising over this range, so the erosion is opposed by the restoring force and is reversible.
- **Regime III (Diversified successor):** $\mu_1 \geq \bar{\mu}_1$. The ordering $x_2^* > \max(x_1^*, x_3^*)$ obtains; the diversified basket holds the plurality and U.S. Treasuries a minority share, second to the basket until gold overtakes them near $\mu_1 \approx 0.114$.

Above the low-disruption threshold of Section 5.5, these regimes are regions of a single, smooth, monotone equilibrium curve, not states separated by a bifurcation. The transition between them is continuous and, by Corollary 1, reversible: a recession of the mutation rate returns the system along the same curve. This is the structural difference from a model with a catastrophic threshold, in which the dominant configuration, once lost, cannot be recovered without rebuilding an advantage from a lower base. Here there is no such trap; the dominant share is a continuous, invertible function of the disruption rate.

5.2 Calibration

The model is calibrated with three asset classes to reproduce the current composition of the global safe asset universe. The baseline parameter values, reported in full in Appendix B and documented empirically in Section 8.1, are $a = (0.935, 0.940, 0.960)$ for intrinsic safety, $b = (1.048, 0.778, 0.663)$ for network-benefit strength, $c = (1.300, 0.980, 1.600)$ for congestion-cost strength, $\gamma = 0.6$ and $\eta = 2.0$ for the two share exponents, and baseline mutation rates $\mu = (0.02, 0.04, 0.03)$. The mutation target matrix τ is calibrated so that mutations away from U.S. Treasuries flow primarily to the diversified basket (55 percent) and secondarily to gold (45 percent), reflecting the substitution patterns documented in the reserve-management literature. The ODE system is integrated numerically using a fourth-order Runge-Kutta method with adaptive step size, imposing a relative

tolerance of 10^{-9} and an absolute tolerance of 10^{-11} . Under these values the dominant equilibrium is $x^* = (0.620, 0.270, 0.110)$: U.S. Treasuries hold approximately 62 percent of the safe asset universe, the diversified basket 27 percent, and gold 11 percent—matching the empirical shares constructed in Section 8.1. The congestion coefficients are ordered $c_3 > c_1 > c_2$, so that gold, the thinnest market, is the most congestion-prone asset and the diversified basket, whose reliance is spread across decorrelated markets, the least. The incumbent carries an intermediate coefficient yet is congestion-dominated in equilibrium through the concentration of reliance, not the size of the coefficient: at the equilibrium its fitness exceeds the population mean ($\sigma = 1.009$) while its marginal fitness is bid just below the basket's.

5.3 Dynamic Transition Under Accelerating Erosion

Figure 6 is the central simulation of the study. It comprises two vertically stacked panels sharing a common time axis. The upper panel is a stacked area chart depicting the market shares of the three asset classes over time. The lower panel displays the mutation rate $\mu_1(t)$, which increases over time according to an accelerating (convex) schedule: $\mu_1(t) = 0.02 + 0.28(t/T)^{1.5}$. This specification is intended to represent the cumulative institutional erosion documented by Auerbach and Gale (2025): not a discrete shock but a gradually accelerating degradation of the fiscal and institutional framework—the financial analogue of the accumulating mutations in the Eigen model.

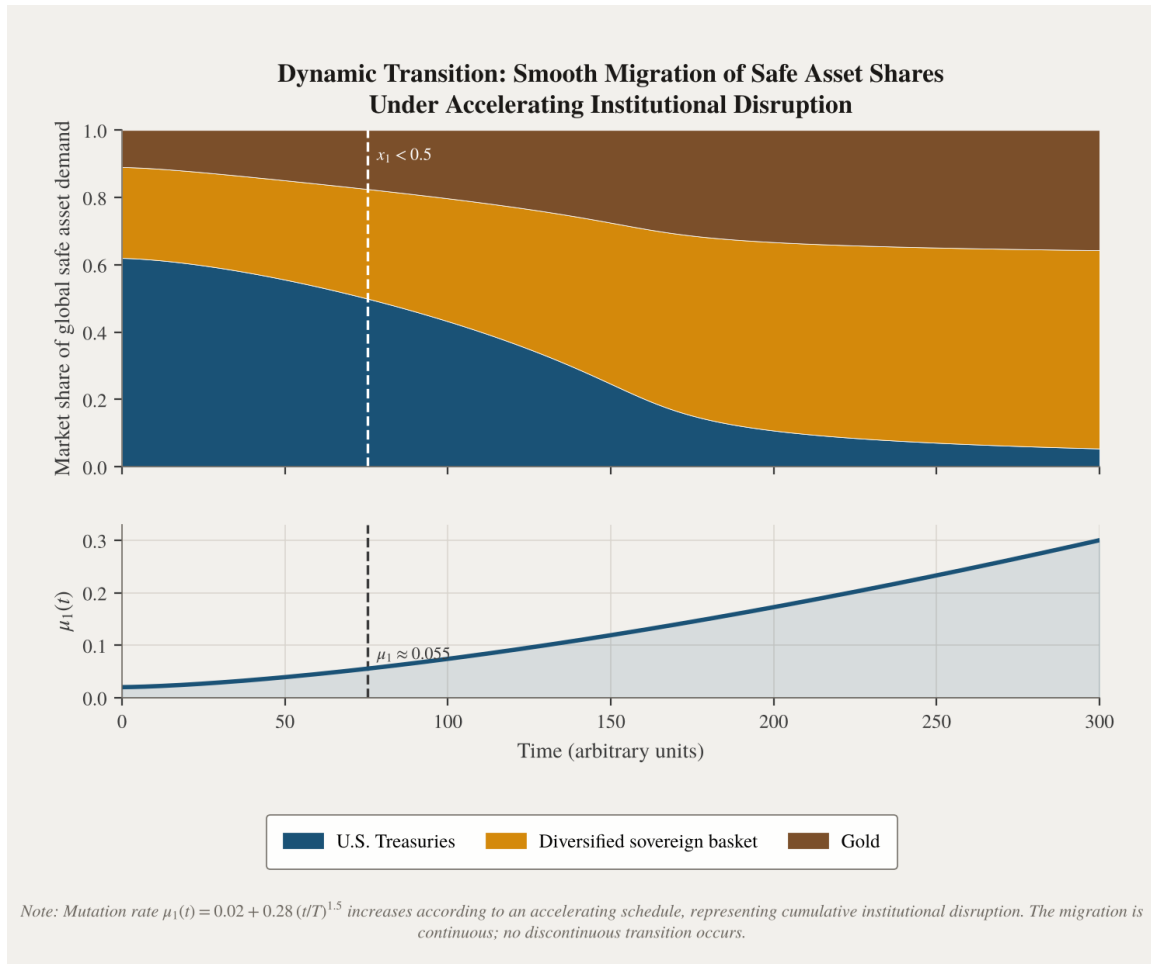


Figure 6. Dynamic Transition Under Accelerating Institutional Erosion

Three regimes, in the sense of Definition 1, are discernible in the upper panel. In Regime I, corresponding to the left portion of the figure, the dark blue area (U.S. Treasuries) holds the majority, occupying roughly three-fifths of the total area, while the diversified basket and gold occupy the remainder. In Regime II, corresponding to the middle portion, the dark blue area contracts continuously as the amber area (diversified basket) and brown area (gold) expand; the dashed white vertical line marks the point at which the U.S. Treasury share falls through 50 percent, which occurs at a mutation rate of approximately 0.055. In Regime III, corresponding to the right portion, the diversified basket overtakes U.S. Treasuries to become the largest single component—a crossing that occurs near $\mu_1 \approx 0.085$ —and at the terminal mutation rate the equilibrium is approximately (0.05, 0.59, 0.36), a diversified configuration in which the basket holds the plurality, gold the second share, and U.S. Treasuries a minority. The migration is continuous throughout; there is no discontinuous collapse. The lower panel depicts the “cause” (the cumulative increase in institutional disruption), the upper panel the “effect” (the gradual reallocation of safe asset demand). By Corollary 1, the trajectory is reversible: a recession of the mutation rate would retrace the same path. The transition is driven not by the crossing of a threshold but by the smooth migration of a unique equilibrium.

5.4 Sensitivity Analysis

Figure 7 provides numerical verification of the comparative statics established in Proposition 3. The figure comprises three panels, each depicting the equilibrium U.S. Treasury market share as a function of the mutation rate μ_1 under variation of a single parameter, with all remaining parameters held at baseline values. Panel A varies the intrinsic fitness a_1 , Panel B the network benefit b_1 , and Panel C the congestion cost c_1 , each at three levels about its baseline. Stronger fundamentals or a stronger network benefit (Panels A and B) raise the equilibrium share and shift the migration curve so that the majority is retained to a higher mutation rate; a larger congestion cost (Panel C) lowers the equilibrium share, since heavier crowding penalties cap the incumbent at a smaller plateau. In every case the curve remains continuous and single-valued—the parameters govern the level and the location of the smooth migration without introducing any discontinuity, consistent with Corollary 1.

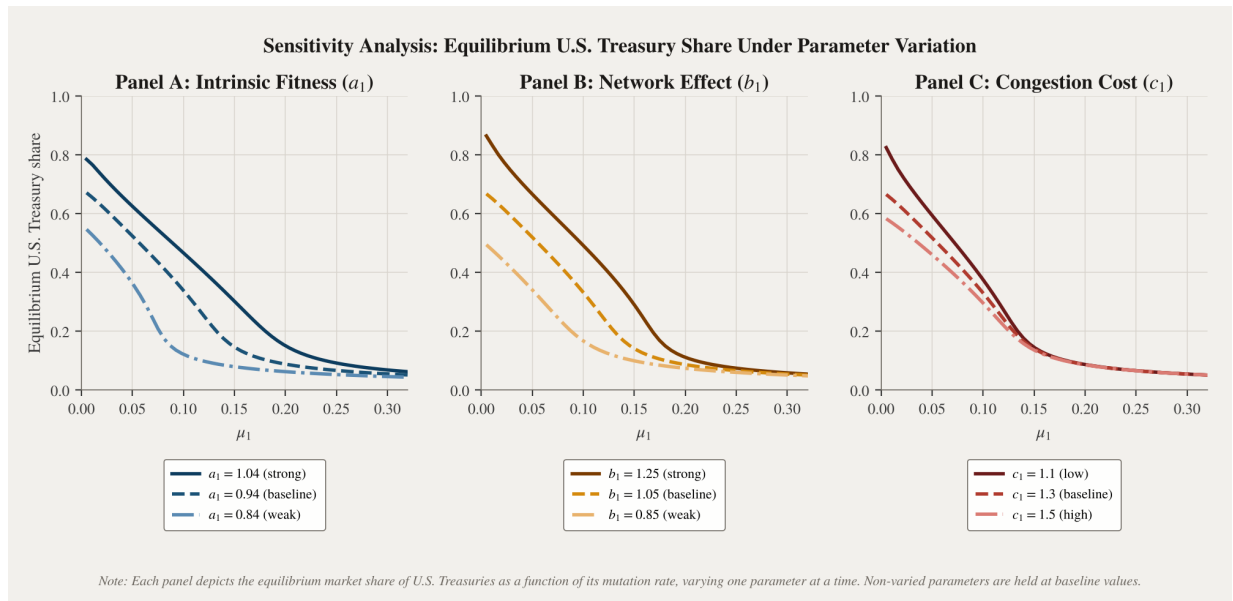


Figure 7. Sensitivity Analysis — Equilibrium U.S. Treasury Share Under Parameter Variation

5.5 Phase Portraits on the Simplex

Figure 8 provides an alternative representation of the dynamics by depicting the flow field of the replicator-mutator system projected onto the two-dimensional simplex. The horizontal axis measures the U.S. Treasury share x_1 , and the vertical axis measures the diversified basket share x_2 ; the gold share $x_3 = 1 - x_1 - x_2$ is determined residually. Arrows indicate the direction and magnitude of the flow at each point; warmer colors signify faster dynamics. Panel A depicts the low-mutation case ($\mu_1 = 0.02$). Trajectories from all depicted initial conditions converge to a fixed point in the lower-right region of the simplex—where x_1 is large and x_2 is small—corresponding to the U.S.-dominant equilibrium of Proposition 1. Panel B depicts the high-mutation case ($\mu_1 = 0.20$). The attractor has migrated toward the interior of the simplex: the equilibrium exhibits a reduced U.S. Treasury share and a more dispersed distribution of market shares across asset classes. In both panels the flow converges to its fixed point; the slowest eigenvalue of the linearized dynamics is

-0.055 at $\mu_1 = 0.02$ and -0.17 at $\mu_1 = 0.20$, and, as Section 7 shows, it is at no rate closer to zero than at the baseline.

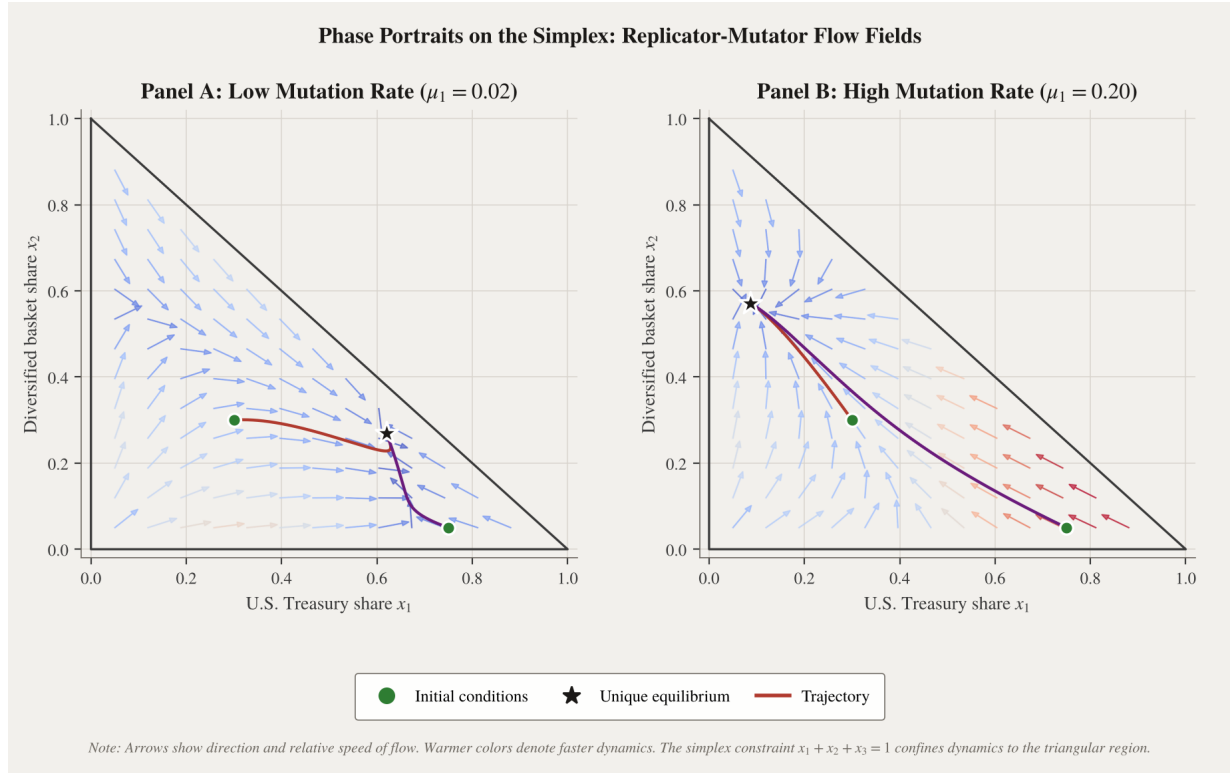


Figure 8. Phase Portraits on the Simplex Under Low and High Mutation Rates

The global character of this attraction at the calibrated rates is robust rather than knife-edge. The statement is scale-free: because multiplying all three fitness components by a common factor only rescales time, leaving the equilibrium and its basins unchanged, absolute coefficient levels carry no meaning, and only relative magnitudes and the mutation rates bear on the basin structure. In the selection limit, as every mutation rate tends to zero, an asset sustains a near-monopoly basin only where its intrinsic-plus-network fitness, net of congestion, exceeds the strongest rival's intrinsic fitness; at the calibration this margin is negative for all three assets, so a near-monopoly basin appears only once congestion weakens, relative to the network and intrinsic terms, to about 0.79 of its calibrated relative strength. Holder turnover lowers the frontier further: at the baseline mutation rates a second basin, the basket's, requires congestion below about 0.29 of that relative strength, so most of the distance to the frontier is supplied by turnover rather than by the congestion level. Variation along the incumbent's own congestion-to-network ratio c_1/b_1 does not cross the frontier at any value of c_1 from 0.05 to 6 (Section 4.5); multiplicity of this kind is a property of a uniformly weak congestion field acting on the alternatives, not of the incumbent's lever. Across 512 perturbations of the calibration within ± 15 percent, each of the nine coefficients of a , b , and c scaled by its own factor, the factors spread over their nine-dimensional box by a deterministic low-discrepancy design, the frontier stays below the calibrated configuration in 511 of the 512 configurations, and in 43 of the 512 no second basin appears at any congestion level examined, from 0.05 of the calibrated strength upward. In the remaining configuration, where the incumbent's intrinsic fitness lies 13 percent below its calibrated value and gold's 14 percent above, two equilibria coexist at its own congestion level, at $(0.10, 0.48, 0.42)$ and $(0.45, 0.11, 0.45)$, the incumbent holding

the majority at neither. At the baseline mutation rates the uniqueness of the equilibrium therefore holds across the defensible parameter set with that one exception, rather than at the calibrated point alone.

Multiplicity of another kind arises at low disruption under the calibrated congestion. Below $\mu_1 \approx 0.0074$ a second stable equilibrium coexists with the dominant one, the incumbent holding the majority in both: at $\mu_1 = 0.005$ the second lies at $(0.72, 0.05, 0.23)$, with gold rather than the basket holding the second share, and the equilibrium of the calibrated branch at $(0.67, 0.27, 0.07)$. The second branch ends at $\mu_1 \approx 0.0074$, where its stable equilibrium merges with a saddle in a saddle-node bifurcation: a system on it would move to the dominant equilibrium as μ_1 rose past that rate and would remain there as μ_1 fell back. The rate lies below the calibrated $\mu_1 = 0.02$, and above it the equilibrium is unique (Corollary 1).

Figure 9 renders two congestion levels at the baseline mutation rates. Under weak congestion the inherited distribution of holdings selects among three near-monopoly basins; at the calibrated configuration a single basin covers the simplex, so the destination is independent of the starting point.

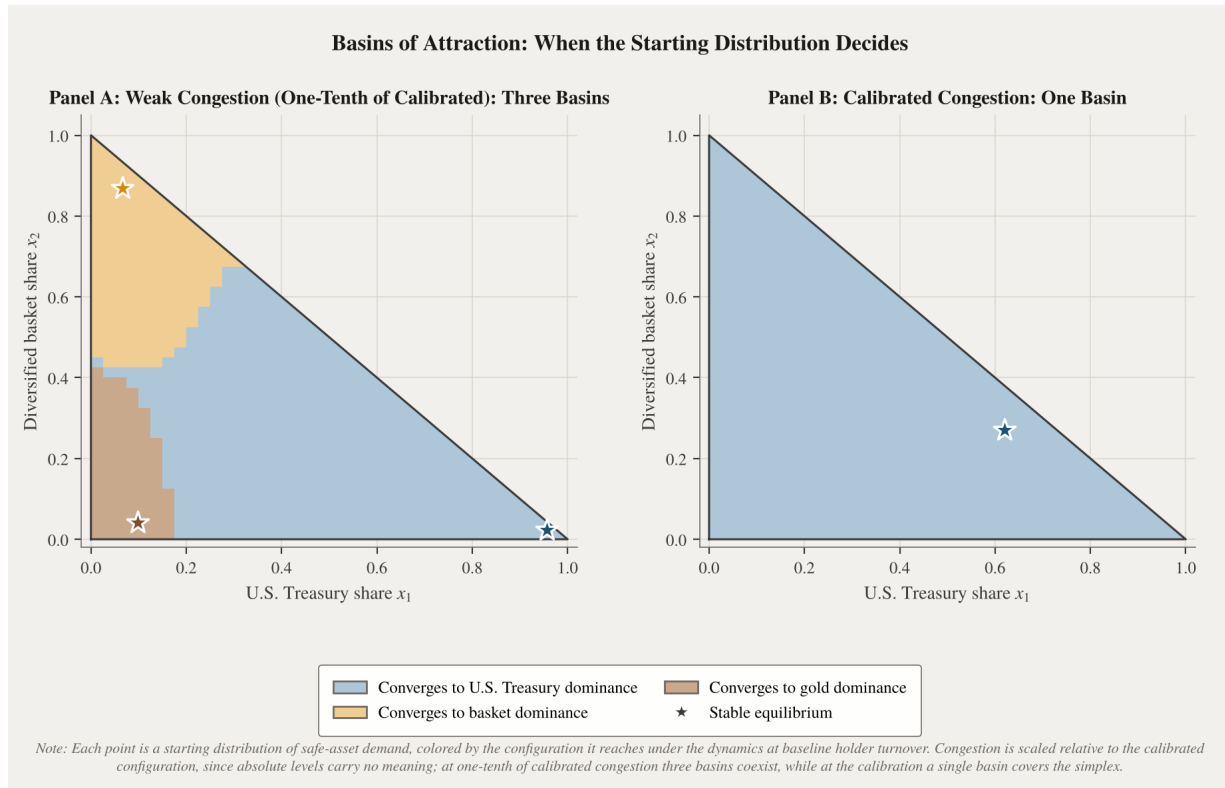


Figure 9. Basins of Attraction Under Weak and Calibrated Congestion

6 The Self-Limiting Feedback Loop

The feature that most distinguishes the congestion formulation both from the classical Eigen quasispecies model and from a purely network-driven account is that the feedback between market share and relative fitness runs in the stabilizing direction. In the Eigen framework the superiority parameter σ is a fixed constant; in a network-effect model it falls as the incumbent's share falls, so erosion compounds. In the present model σ is a function of the equilibrium share through both the network benefit and the congestion cost,

$$\sigma(x_1^*) = \frac{f_1(x_1^*)}{\phi(x^*)} = \frac{a_1 + b_1(x_1^*)^\gamma - c_1(x_1^*)^\eta}{\sum_i x_i^* [a_i + b_i(x_i^*)^\gamma - c_i(x_i^*)^\eta]},$$

and on the dominant-equilibrium branch it rises as the incumbent's share falls, past the loss of the majority to its maximum near $\mu_1 = 0.112$ (Proposition 2 and Appendix A).

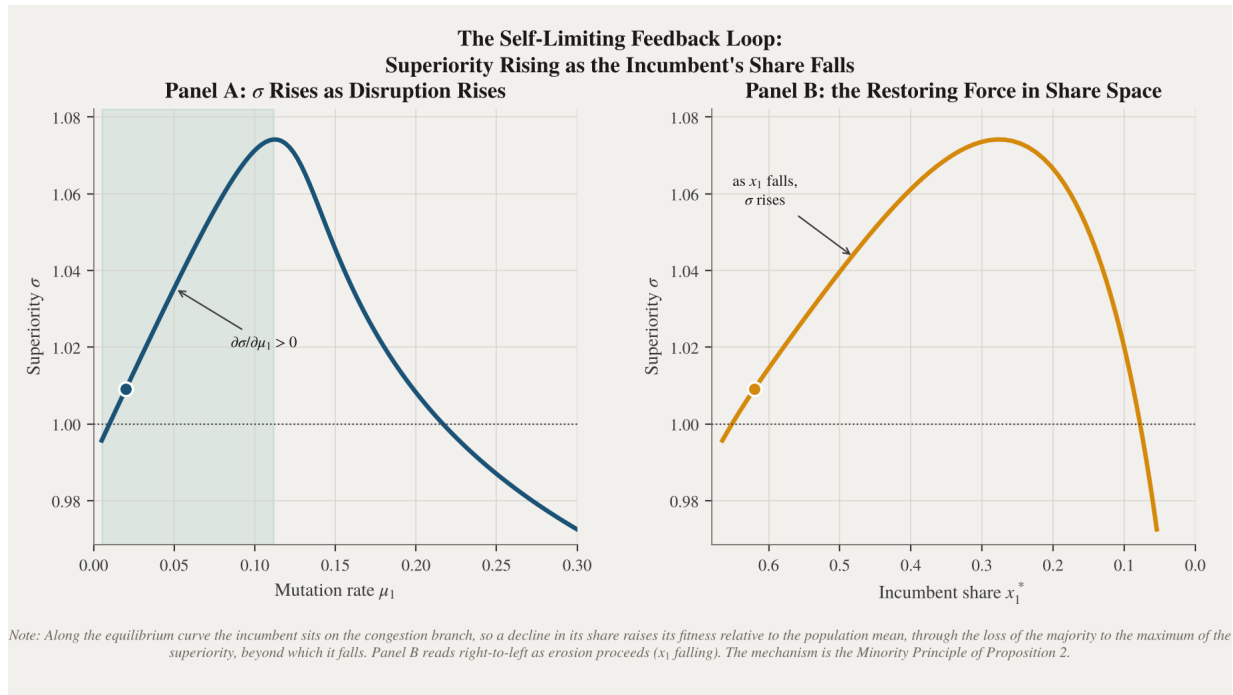


Figure 10. The Self-Limiting Feedback Loop — Superiority Rising as the Incumbent's Share Falls

Figure 10 renders the mechanism. Panel A plots the superiority parameter σ along the equilibrium curve as a function of the mutation rate. Beginning from the calibrated equilibrium, σ rises as μ_1 increases and the incumbent's share declines, because the dominant share sits on the congestion-dominated branch of the fitness profile: a reduction in share relieves congestion faster than it surrenders network benefit, raising the incumbent's fitness relative to the population mean. Panel B traces the same relationship against the incumbent's share directly, showing σ falling as x_1 rises toward saturation—the formal statement that excessive concentration is self-defeating.

This generates a negative, stabilizing feedback loop: institutional disruption reduces the incumbent's share, which relieves congestion, which raises the incumbent's relative fitness, which generates a restoring force opposing further erosion. The dominant configuration is held in place not by an unassailable advantage but by the congestion that caps it: the asset is most attractive at

the margin when reliance upon it has thinned. This is the safe asset paradox stated as a stabilizing principle—the Minority Principle—and it is the mechanism the empirical literature has reached for, without formalizing, in accounting for the episodes of partial recovery in the dollar’s reserve share. It stands in deliberate contrast to the self-undermining account of Brunnermeier, Merkel, and Sannikov (2024) and Gorton (2017): safe asset status, in the present model, erodes only so far as disruption compels and recovers as disruption recedes. This self-limiting property is specific to the disruption channel: a structural increase in the incumbent’s congestion cost—a permanent loss of absorption capacity rather than a transient disruption—is not offset by the restoring force (Section 4.5), and the recovery described here follows the recession of disruption, not the reversal of a structural deterioration.

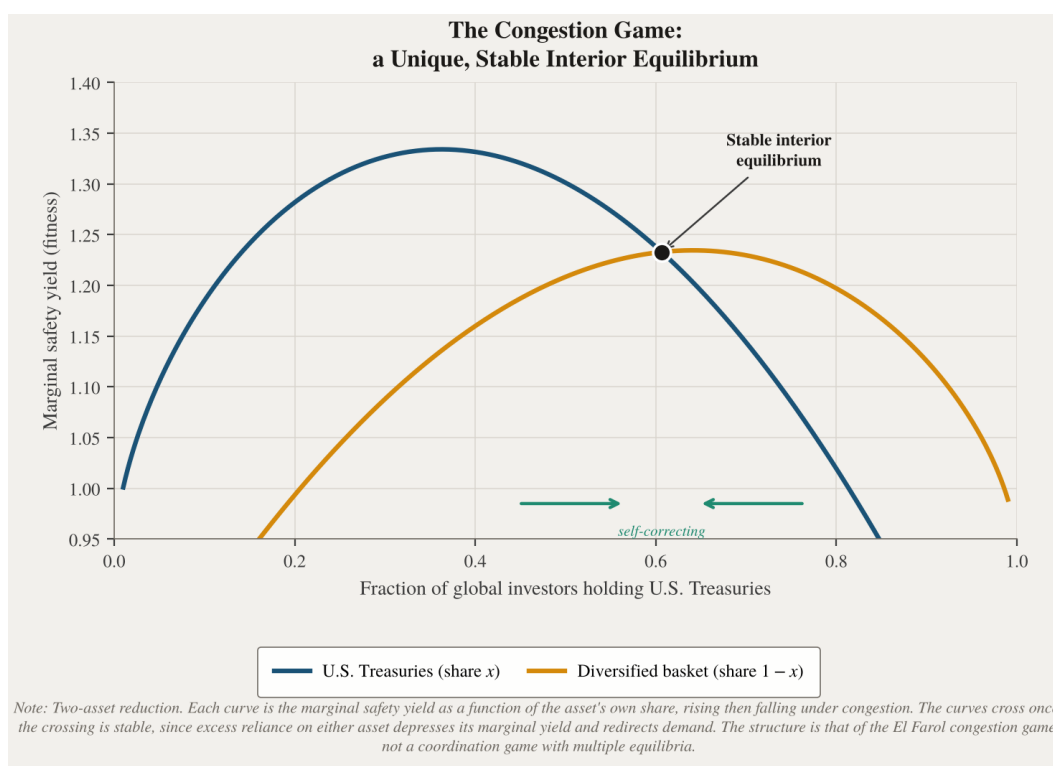


Figure 11. The Congestion Game: a Unique Stable Interior Equilibrium

Figure 11 situates the result against the game-theoretic structure of safe asset choice. The horizontal axis measures the fraction of global investors holding U.S. Treasuries as their safe asset. The marginal safety yield is rising at low shares, where liquidity depth accrues, and falling at high shares, where congestion degrades the asset’s reliability under stress; the indifference point at which it equals the alternative’s yield is a stable interior equilibrium, indicated by the black dot. This is the structure of a congestion game in the sense of the El Farol problem, not of a coordination game with multiple equilibria: there is a single attracting configuration, and deviations from it—too many or too few holders—are self-correcting. The bistable, tipping-prone landscape of the network-effect account is replaced by a single, congestion-anchored resting point.

7 The Absence of Critical Slowing Down

The self-limiting feedback loop of the preceding section has a testable corollary for the statistics of the state variable. A model with a catastrophic threshold makes the opposite prediction, and the contrast furnishes an empirical test that discriminates between the two accounts. It is well established that the approach to a saddle-node bifurcation is accompanied by Critical Slowing Down (CSD): the dominant eigenvalue of the linearized dynamics approaches zero, the restoring force weakens, perturbations decay ever more slowly, and the variance and lag-one autocorrelation of the state variable rise toward their limiting values (Scheffer, Bascompte, Brock, Brovkin, Carpenter, Dakos, Held, van Nes, Rietkerk, and Sugihara 2009). Rising variance and rising autocorrelation are the canonical early-warning signatures of an impending regime shift.

In the congestion model, by contrast, the restoring force never vanishes along the dominant branch. Above the low-disruption threshold of Section 5.5, which lies below the calibrated rate, the equilibrium is unique and varies smoothly with the disruption rate (Proposition 3 and Corollary 1), so there is no bifurcation for the dynamics to approach, and the dominant eigenvalue does not approach zero. Computed along the equilibrium curve, the dominant eigenvalue, the slowest of the linearized system, remains bounded away from zero throughout the transition: it is closest to zero—weakest restoring force—near the baseline majority equilibrium (-0.055 at $\mu_1 = 0.02$). It is not monotone in the disruption rate: it reaches -0.13 near $\mu_1 = 0.08$, rises to -0.065 near $\mu_1 = 0.13$, and reaches -0.30 at $\mu_1 = 0.30$, at no rate of the range closer to zero than at the baseline.

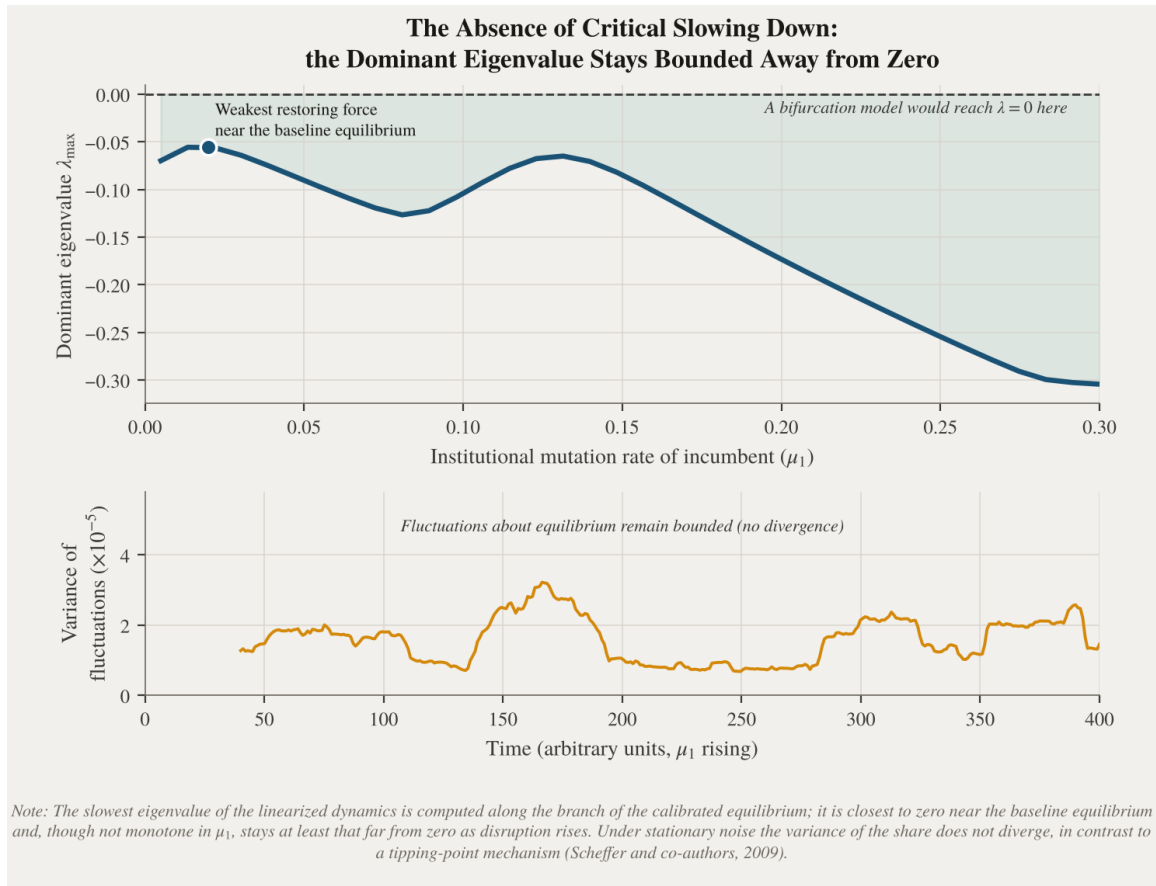


Figure 12. Restoring Force Along the Transition: the Dominant Eigenvalue Remains Bounded Away from Zero

Figure 12 displays the dominant eigenvalue of the linearized dynamics as a function of the mutation rate, with a stochastic realization of the share for reference. In a bifurcation model the eigenvalue would rise to zero at the threshold and the variance of the share would diverge; here the eigenvalue stays strictly negative and the variance remains bounded across the entire range. The model therefore predicts that the canonical early-warning indicators—rising variance and rising autocorrelation in Treasury yield spreads, repo haircuts, or recovery times following political shocks—will not exhibit a systematic divergence as the dollar’s safe asset share erodes. The absence of such signatures, which a tipping-point mechanism would generate, is the empirical fingerprint of a smooth, self-limiting transition. This prediction is consistent with the observation, noted in the empirical literature, that the secular decline in the dollar’s reserve share has proceeded without the volatility clustering that would precede a discontinuous regime shift.

8 Empirical Calibration

8.1 The Calibration Target: the Global Safe Asset Universe

The preceding sections have developed the model as a theoretical framework. This section anchors it empirically, in two steps: first by constructing the calibration target—the observed composition of the global safe asset universe, to which the equilibrium x^* is matched—and then by mapping the remaining parameters onto observable proxies. The exercise is illustrative rather than a formal econometric estimation, since several mappings require judgment; its purpose is to establish that credible proxies exist and that the calibrated model reproduces the current configuration.

The state variable x_1 is the U.S. Treasury share of the global safe asset universe. That universe is defined here as the stock of marketable safe sovereign debt—obligations rated Aa3/AA– and above, the Basel high-quality-liquid-asset Level 1 tier carrying a zero risk weight—together with monetary gold, measured across all holders. The definition is deliberately confined to the sovereign layer and gold. Privately produced safe assets, principally agency mortgage-backed securities and highly rated corporate paper, are excluded: they constitute a within-currency supply layer downstream of the sovereign choice, and including dollar-denominated private safe assets would conflate the Treasury-versus-alternatives question that the model addresses with the separate question of dollar-denominated supply. The definition thus preserves the distinction, maintained throughout, between the safe asset (Treasury) and the currency of denomination (the dollar).

On this definition, the universe at the end of 2025 comprises approximately the following, in trillions of U.S. dollars. U.S. Treasuries account for about 30.3 (marketable Treasury debt, per the Securities Industry and Financial Markets Association and the Federal Reserve’s financial accounts). The diversified basket of other safe sovereigns accounts for about 13.2: German Bunds (2.5), French obligations (3.0), United Kingdom gilts (3.4), and smaller amounts of Dutch, Canadian, Australian, and Swiss paper, together with highly rated supranational issues (about 2.0). Monetary gold held in official reserves accounts for about 5.2 (some 36,500 tonnes valued at end-2025 prices, per the World Gold Council and the International Monetary Fund). The total is approximately 48.7, yielding shares of about 62, 27, and 11 percent for Treasuries, the basket, and gold respectively—the values to which the model is calibrated, $x^* = (0.620, 0.270, 0.110)$.

Two sovereign exclusions warrant comment. Japanese government bonds, though large (about 7.8 trillion), are omitted because their A+ rating falls below the Level 1 threshold and because they are overwhelmingly domestically held and do not function as a global safe asset in the sense the model requires. Euro-area periphery debt (Italian, Spanish) is excluded on the rating criterion. The target is not knife-edge: restricting the basket to AAA issuers raises the Treasury share to about 72 percent (close to the dollar’s 1999 reserve-share peak), while measuring Treasuries net of Federal Reserve holdings or counting privately held investment gold lowers it toward 57–59 percent. The defensible range is therefore approximately 59 to 72 percent, centered near 62 (Section 8.3 compares it with the calibrated equilibrium curve).

The repurchase-agreement market corroborates this picture from the demand side, in use rather than in stock. The Office of Financial Research reports that the U.S. repo market averaged about 12.6 trillion in daily exposures in the third quarter of 2025, of which U.S. Treasuries collateralized about 69 percent overall and about 89 percent in the centrally cleared segment. These figures weight safe assets by their mobilization as collateral in the core funding market and confirm that Treasury dominance in the safe asset function is, if anything, more pronounced than the stock measure indicates; the cross-currency composition of bilateral repo—dollar foremost, then

euro, sterling, yen, and Canadian dollar—mirrors the cross-sovereign ordering of the basket. The repo evidence also supplies the standing capacity constraint that underlies the congestion term: a market organized around netting and zero-haircut packages to economize on scarce dealer balance sheet is direct evidence that the capacity to intermediate the safe asset is finite, the precondition for the safety degradation that the congestion cost represents. The repo gradient across clearing venues is not used here as evidence of congestion, since it is governed by collateral-eligibility rules and balance-sheet economics rather than by a sorting of safety demand.

8.2 Parameter Mapping

Beyond the share x_1 , the model requires empirical proxies for the superiority parameter σ , the mutation rate μ_1 , and the complexity parameter L . The following paragraphs describe the proxy selected for each and the rationale for the selection.

For the superiority parameter σ , the proxy is the convenience yield on U.S. Treasury securities. The literature offers at least four measurement approaches. The first and longest-running is the spread between Moody’s Aaa corporate bond yield and the yield on 10-year Treasury bonds, established as the standard measure by Krishnamurthy and Vissing-Jorgensen (2012), who showed that this spread is a decreasing function of the outstanding supply of Treasuries relative to GDP and averages approximately 73 basis points over their sample. The second is the TIPS-Treasury premium of Fleckenstein, Longstaff, and Lustig (2014), which compares the yield on a nominal Treasury with the yield on a synthetic nominal bond constructed from Treasury Inflation-Protected Securities (TIPS) and inflation swaps; because both legs are issued by the same sovereign, differences in credit risk are eliminated. The third is the U.S. Treasury Premium of Du, Im, and Schreger (2018), which measures the gap between the foreign exchange swap-implied dollar yield paid by near-default-free foreign governments and the U.S. Treasury dollar yield; this captures the convenience yield of U.S. Treasuries relative to the bonds of other advanced sovereigns. The fourth is the box-rate construction of van Binsbergen, Diamond, and Grotteria (2022). A recent Federal Reserve Board paper by Du and Schreger (2025) extends the third measure through June 2025 and decomposes it into a dollar convenience component and a Treasury-specific convenience component.

For the present calibration, the Aaa-Treasury spread is adopted as the primary proxy for σ owing to its length (available from 1919), its wide use in the literature, and its availability on the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of St. Louis. The mapping is $\sigma = 1 + \kappa \cdot CY$, where CY is the Aaa-Treasury spread in percentage points and κ is a scaling constant. Figure 13 displays the first three of these measures side by side. The measures co-move at business-cycle frequencies—all spike during the Global Financial Crisis (GFC) and COVID-19—but diverge at lower frequencies. In particular, the Du-Im-Schreger five-year premium has turned negative in the post-crisis period, indicating that the United States no longer enjoys a convenience premium over other advanced sovereigns at medium maturities. In the congestion model this is not a sign of imminent collapse but the expected empirical counterpart of the equilibrium: with the incumbent’s share on the congestion-dominated branch, its marginal fitness is bid down to, and slightly below, the basket’s (Proposition 1, $f_2 > f_1$), so the superiority parameter sits just above unity ($\sigma \approx 1.01$ at the calibrated equilibrium) and the measured premium over comparable sovereigns is small or negative. A convenience yield near zero is thus consistent with a stable majority equilibrium, not evidence of erosion past a threshold; what the model reads from the secular compression of the premium is that the system is at a congested interior equilibrium, the regime in which the restoring force of Proposition 2 operates.

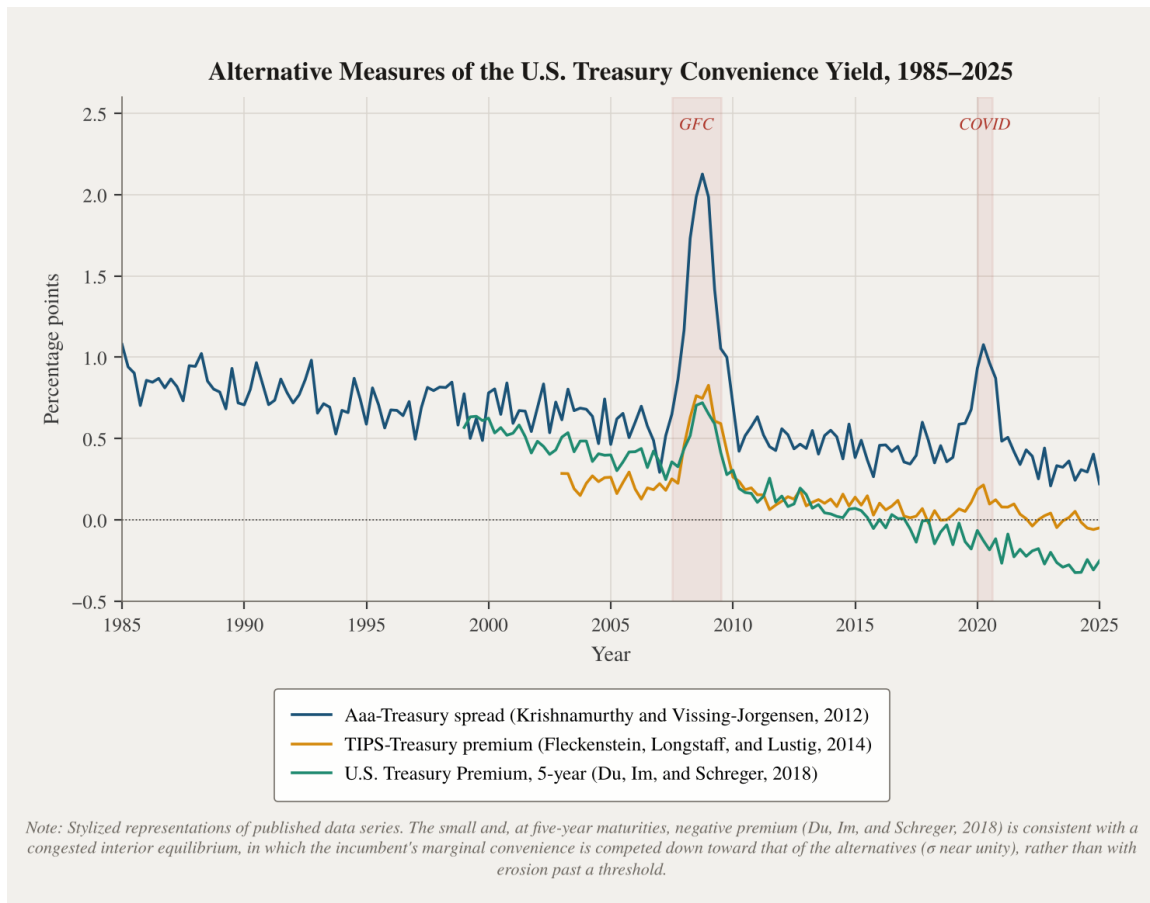


Figure 13. Alternative Measures of the U.S. Treasury Convenience Yield, 1985–2025

For the mutation rate μ_1 , the primary proxy is the Baker, Bloom, and Davis (2016) Economic Policy Uncertainty (EPU) index, available at monthly and daily frequency on FRED from January 1985 to the present. This index spikes at the events the model classifies as institutional mutations: debt-ceiling confrontations (2011, 2013, 2023), government shutdowns, the September 11 attacks, the Lehman Brothers failure, and major fiscal policy reversals. Three alternative measures capture complementary facets of institutional disruption. The first is the IMF Fiscal Policy Uncertainty (FPU) index (Furceri, Ganslmeier, Ostry, and Yang 2024), constructed for 189 countries using Dow Jones Factiva archives. The second is the Caldara and Iacoviello (2022) Geopolitical Risk (GPR) index, published in the *American Economic Review* and available on FRED from 1985. The third is the Caldara, Iacoviello, Molliigo, Prestipino, and Raffo (2020) Trade Policy Uncertainty (TPU) index, which the Federal Reserve Board highlighted in its April 2025 note “Costs of Rising Uncertainty” as having reached its highest levels in decades. A cross-check is provided by the Atlanta Fed Survey of Business Uncertainty (SBU), which measures uncertainty directly from firm-level sales forecasts rather than from newspaper text, and which—as Bloom (2025) observes in *Finance and Development*—has not spiked in the current period, raising the question of whether text-based measures overstate actual uncertainty.

For the complexity parameter L , the primary proxy is the QuantGov/RegData restriction count developed by McLaughlin and Al-Ubaydli at the Mercatus Center (George Mason University). Rather than counting pages in the Code of Federal Regulations (CFR), RegData counts binding

regulatory restrictions: words such as “shall,” “must,” “may not,” and “required” within the regulatory text. This count grew from approximately 830,000 restrictions in 1970 to over one million by the 2010s. Version 3.2 (covering 1970–2019) also provides complexity metrics that are apt as proxies for L : the number of conditional terms or phrases, average sentence length, reading grade level, and Shannon Entropy of the regulatory text. The Shannon Entropy measure is the closest analog to the genome length in the Eigen model. A second proxy isolates the financial-sector regulatory burden specifically: the RegData industry-specific restriction count for North American Industry Classification System (NAICS) sector 52 (Finance and Insurance). A third proxy captures the sanctions-weaponization dimension: a composite measure combining the Office of Foreign Assets Control (OFAC) Specially Designated Nationals and Blocked Persons (SDN) list count, the number of active U.S. sanctions programs, and the number of jurisdictions subject to comprehensive sanctions. The CFR total page count, compiled annually by the George Washington University Regulatory Studies Center (RegStats), remains available as the longest and most widely cited series, but it is the least precise of the available proxies.

For the market share x_1 , the primary anchor is the U.S. Treasury share of the global safe asset universe constructed in Section 8.1, approximately 62 percent. The U.S. dollar’s share of disclosed global foreign exchange reserves, reported in the International Monetary Fund’s Currency Composition of Official Foreign Exchange Reserves (COFER) database, provides a corroborating reserve-side dimension: it has declined from approximately 71 percent in 2001 to approximately 56 percent in mid-2025, though, following the IMF’s own guidance, much of the recent decline reflects exchange-rate valuation effects rather than active portfolio rebalancing. The COFER series measures the currency of reserves rather than the safe asset directly, and so sits below the universe share, which counts the Treasury stock against other safe sovereign debt and gold. Four further measures capture complementary dimensions of the dollar’s footprint: the share of foreign exchange transactions from the Bank for International Settlements (BIS) Triennial Central Bank Survey (April 2025: 89 percent, unchanged from 2022 and 2019); the share of international debt securities outstanding (over 60 percent); the share of international payment messages of the Society for Worldwide Interbank Financial Telecommunication (SWIFT) (approximately 47–49 percent); and the Federal Reserve Board’s composite Index of International Currency Usage (in the 65–70 range). The salient feature of the comparison is divergence: the reserve share has declined while the transaction share and the composite index have been stable. This is itself informative: erosion is concentrated in the store-of-value dimension while the medium-of-exchange dimension remains entrenched, a pattern consistent with the Gopinath and Stein (2021) finding that invoicing complementarities create lock-in, and a reminder that the single-instrument calibration abstracts from the dollar’s multidimensional footprint.

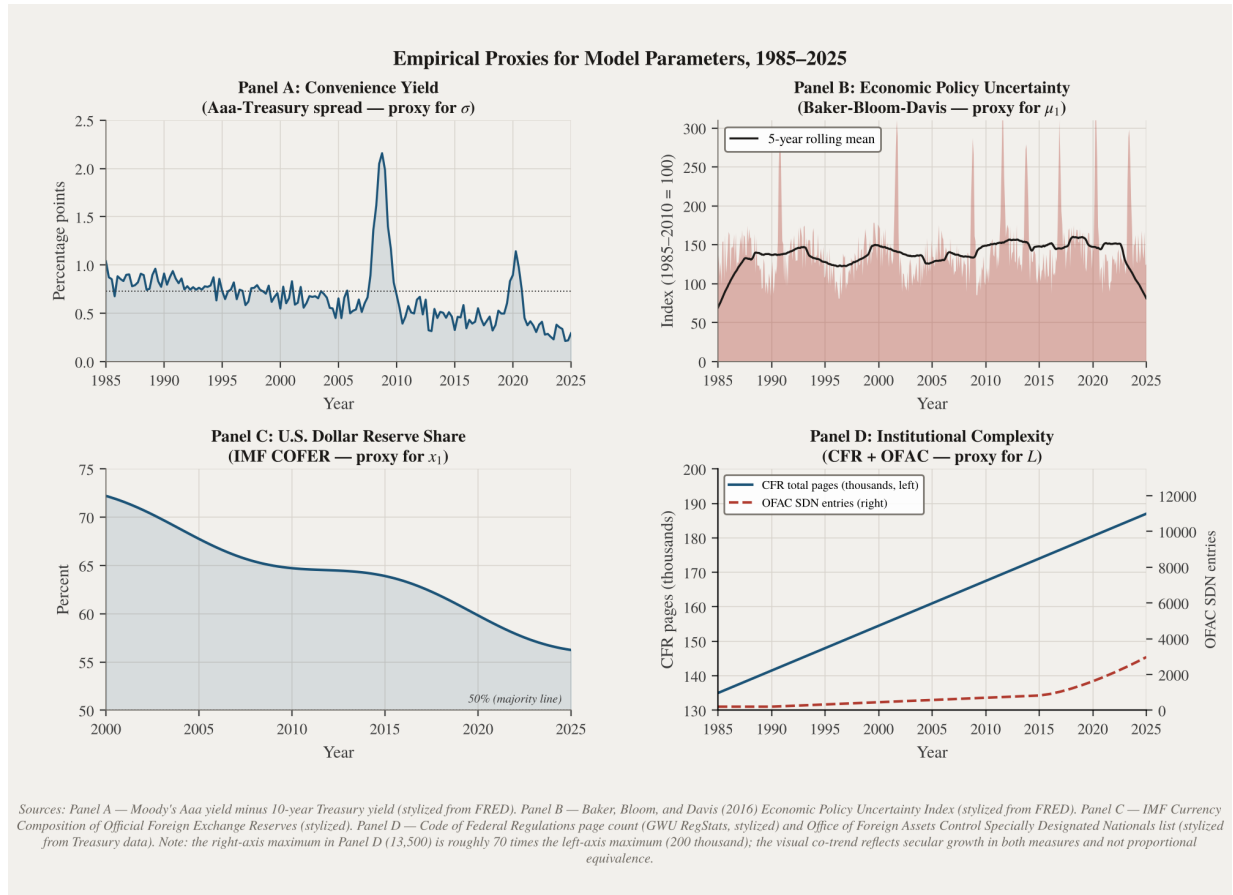


Figure 14. Empirical Proxies for Model Parameters, 1985–2025

8.3 Locating the System on the Equilibrium Curve

Because the model has no error threshold, the empirical exercise is not to compute a distance to a catastrophe but to locate the system on the smooth equilibrium curve and to characterize the forces acting on it. The inversion is direct: the observed U.S. Treasury share of the safe asset universe, approximately 62 percent, corresponds on the calibrated equilibrium curve to a low baseline disruption rate, $\mu_1 \approx 0.02$, placing the system in Regime I, where Treasuries hold a stable majority. The loss of the majority ($x_1^* = 1/2$) would require the disruption rate to roughly treble, to $\mu_1 \approx 0.055$; the basket would assume the plurality only near $\mu_1 \approx 0.085$. The observed position is therefore inside the majority regime, not at the edge of a transition.

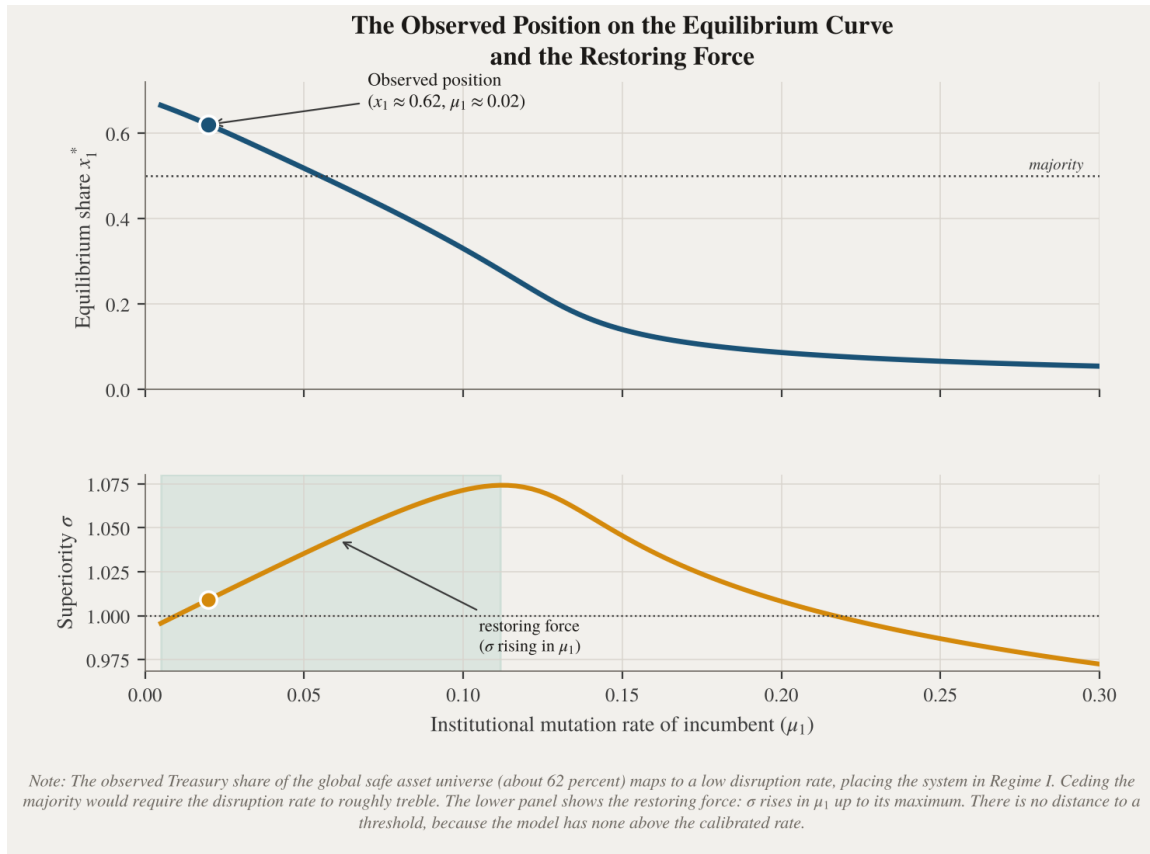


Figure 15. The Observed Position on the Equilibrium Curve and the Restoring Force

Figure 15 places the observed configuration on the equilibrium curve and shows the restoring force acting at that point. The upper panel marks the current share against the full curve $x_1^*(\mu_1)$; the lower panel plots the superiority σ , which is rising in μ_1 up to its maximum near $\mu_1 = 0.112$ (Proposition 2). Two features of the empirical record are accommodated by this reading that a threshold model cannot accommodate. First, the secular decline in the convenience yield does not signal approach to a cliff: at a congested interior equilibrium the marginal premium is small by construction, and its compression is the signature of the equilibrium rather than of erosion past a boundary. Second, the episodes of partial recovery in the dollar's reserve share—a decline followed by a renewed rise—are the expected response to a recession of disruption, because the restoring force operates symmetrically along the reversible curve.

The interpretation is necessarily qualitative. The scaling constants mapping the Aaa-Treasury spread into σ and the EPU index into μ_1 involve judgment, and the safe-asset-universe share is itself a constructed quantity with a defensible range of 59 to 72 percent. On the calibrated curve the incumbent's share approaches about 0.68 on the dominant branch as μ_1 approaches zero, so the upper part of that range lies above the branch; shares up to about 0.73 are equilibria only on the second, low-disruption branch of Section 5.5, at which gold rather than the basket holds the second share. What the calibration establishes is not a precise coordinate but a regime: the system rests at a congestion-stabilized majority equilibrium, subject to a restoring force, with room before the majority would be ceded and no threshold beyond which recovery becomes impossible.

8.4 Consistency with the Eichengreen (2011) Thesis

The empirical calibration bears directly on the thesis advanced by Eichengreen (2011), who argued that the dollar would lose its singular status gradually rather than suddenly, and that the transition would yield a multipolar configuration rather than a new unipolar system. An elaboration of this thesis is provided by Chitu, Eichengreen, and Mehl (2014), whose historical analysis of the interwar period demonstrates that sterling and the dollar coexisted as co-equal reserve currencies through the 1920s—with the dollar surpassing sterling in identified reserves as early as the mid-1920s, well before the Bretton Woods arrangements of 1944 institutionalized dollar primacy. Their finding complicates the “incumbent-until-suddenly-displaced” narrative that a simpler reading of the Schmelzing (2020) data might support: reserve-currency status can be shared for extended intervals, and the gradual migration of the present model—a monotonic decline in the incumbent’s share, accompanied by the rise of alternatives—is consistent with such a period of shared status.

The model’s regime dynamics—gradual erosion of market share as μ_1 rises, without a discrete break—are therefore consistent with both the Eichengreen narrative and the Chitu-Eichengreen-Mehl historical evidence. The secular decline in the COFER dollar share from 71 to 56 percent over a quarter-century is the pattern that the smooth migration generates, and the shared-status interval that Chitu, Eichengreen, and Mehl (2014) document corresponds to the model’s interior configuration, in which several safe assets coexist. The model sharpens Eichengreen’s narrative in a direction that strengthens its gradualism rather than qualifying it. Where the rational-bubble and network-effect accounts hold that a gradual decline must eventually give way to a self-reinforcing collapse, the congestion mechanism implies the opposite: erosion is self-limiting, the equilibrium curve is reversible, and there is no such boundary to cross. The transition is gradual not merely until a threshold but throughout the range above the low-disruption threshold of Section 5.5, because there the equilibrium is unique and moves continuously (Corollary 1), and σ rises as the incumbent’s share declines up to its maximum (Appendix A). The Eichengreen gradualism is thus not a prelude to a discontinuous break but the entire character of the process.

8.5 Tensions Between Model and Data

An assessment of the calibration requires an explicit accounting of where the data sit in tension with the model. The reformulation around congestion resolves two of the tensions that a network-effect or classical quasispecies version confronts, leaves one in modified form, and renders a fourth largely moot.

The tension it resolves most directly concerns the episodes of partial recovery in the dollar’s reserve share. After declining from approximately 66 percent in 2003 to 61 percent in 2011, the COFER dollar share recovered to approximately 66 percent by 2015 before resuming its secular decline. A self-reinforcing mechanism predicts that erosion begets erosion and cannot accommodate such reversals; the congestion model predicts them. By Proposition 2 the restoring force operates symmetrically along a reversible equilibrium curve, so a decline induced by elevated disruption is recovered when disruption recedes. The reversals are not anomalies to be explained away but the signature of the self-limiting mechanism.

The second tension it resolves concerns the proximity to instability. A threshold model implies that the system stands a finite, and possibly small, distance from a catastrophe, with the distance sensitive to the complexity parameter L . The congestion model has no such boundary: the equilibrium is unique at the calibration, the restoring force does not vanish, and critical slowing down does

not occur (Section 7). The model therefore does not overstate the proximity to collapse, because it posits no collapse to be proximate to. Relatedly, this removes the model’s former acute sensitivity to L . Because L entered only through the error-threshold formula, which no longer exists, the mapping from the regulatory page count to institutional fragility—a genuine difficulty, since some regulation strengthens rather than weakens the safe asset property—no longer drives a knife-edge conclusion. The complexity proxy remains a coarse instrument, but little now turns on it.

The tension that remains, in modified form, is the counter-cyclical behavior of the convenience yield during crises. During the Global Financial Crisis and the COVID-19 pandemic the Aaa-Treasury spread widened: investors fled into Treasuries, reinforcing their safe asset status at the moment of maximum stress—the “exorbitant duty” of Gourinchas and Rey (2007). The congestion model supplies a structural restoring force, but not this crisis-specific one: its fitness functions are stationary, so they do not shift upward in adverse states of the world. The model thus captures the secular restoring force (congestion relief as share erodes) but not the cyclical one (flight to safety in crises). The two are distinct mechanisms, and only the former is endogenous to the present model; the latter is the leading candidate for extension.

A measurement tension also persists. The IMF’s own analysis of the 2025 data shows that, once exchange-rate movements are accounted for, the reserve-share decline was a small fraction of the headline figure. This bears less on the present calibration than on a reserve-share calibration, because x_1 is anchored here to the safe-asset-universe stock rather than to the COFER currency share; but it remains relevant to the inference of μ_1 from the pace of observed reallocation.

8.6 Assessment: What the Calibration Establishes and What It Does Not

The calibration establishes more than co-movement. The model reproduces the level of the current configuration—the Treasury, basket, and gold shares of the safe asset universe—as a stable equilibrium, not merely the direction of recent changes. It accommodates the partial recoveries that a self-reinforcing account cannot. And it makes a distinctive, falsifiable prediction that separates it from a tipping-point model: the absence of critical slowing down: variance and autocorrelation in the relevant spreads should not diverge as the share erodes. The empirical record, in which the secular decline has proceeded without volatility clustering of the kind that precedes a discontinuous shift, is consistent with this prediction.

What the calibration does not establish is a unique structural identification. The observed trajectory is also consistent with a reduced-form mean-reverting process with a slowly drifting central value; the congestion model is, in one sense, such a process endowed with economic content—the central value is the congestion-stabilized equilibrium, and the reversion is the Minority Principle restoring force. The model’s claim to distinction rests on that content and on its specific predictions: the sign of $\partial\sigma/\partial\mu_1$, the identity of the diversified successor, and the absence of early-warning signatures. Nor does the calibration capture the crisis-time flight-to-safety, which remains the one qualitative feature of the data the stationary specification omits.

8.7 Directions for Model Extension

The congestion mechanism that earlier treatments proposed as a future extension is, in the present model, part of the core specification. The tensions that remain suggest three further directions.

The first would introduce state-dependent fitness to capture the crisis-time flight to safety. Let $s_t \in \{normal, stress\}$ denote the state of the global economy, identified by a threshold on an observable stress indicator S_t (for example, the Chicago Board Options Exchange Volatility Index, or the Gilchrist-Zakrajsek Excess Bond Premium), and let the intrinsic fitness of U.S. Treasuries be

$$a_1(s_t) = \bar{a}_1 + \lambda \cdot \mathbf{1}\{s_t = stress\}, \quad (5)$$

with $\lambda > 0$ capturing the magnitude of the flight-to-safety effect. Under this extension the incumbent's fitness, and hence its equilibrium share, would rise during crises, generating the counter-cyclical reinforcement the data exhibit while leaving the secular congestion dynamics intact. The cyclical restoring force of the flight to safety and the secular restoring force of congestion relief would then operate together. A calibration sets $\bar{S}$ at the ninetieth percentile of the stress indicator and λ so that the implied convenience-yield spike at the 2008 peak matches the observed spike of roughly 1.5 percentage points.

The second extension would incorporate the Gopinath and Stein (2021) trade-invoicing complementarity, embedding the invoicing share as a state variable that co-evolves with the convenience yield—as developed in Schmid (2026)—to reproduce the divergence between the stable transaction share and the declining reserve share. The third would introduce heterogeneous agents: central banks, sovereign wealth funds, and large institutional investors are strategically distinct players whose portfolio decisions carry disproportionate weight. Embedding the dynamics in a game with heterogeneous, strategically interacting holders—in the spirit of He, Krishnamurthy, and Milbradt (2019)—would connect the congestion mechanism to the holder heterogeneity that determines how concentrated reliance becomes, and hence how binding the congestion cost is.

9 Implications for the United States

9.1 The Disruption Rate and the Erosion of Share

The fiscal trajectory of the United States, as documented by the Congressional Budget Office (CBO) and analyzed by Auerbach and Gale (2025), represents a sustained intensification of the conditions the model identifies with a rising mutation rate μ_1 . Federal debt held by the public stands at approximately 100 percent of GDP in fiscal year 2025 and is projected to attain 156 percent by 2055 under the CBO baseline, or 175 percent by 2056 under the extended projections of the Committee for a Responsible Federal Budget (CRFB). Net interest payments are projected to rise from 3.3 percent of GDP in 2026 to approximately 7 percent by 2056, surpassing all discretionary expenditure by 2038. Concurrently, the institutional complexity of the U.S. financial and legal system has been augmented through the proliferation of sanctions regimes and contingent liabilities, while the convenience premium has compressed as other economies have grown relative to the United States.

In the present model these trends raise μ_1 and, by Proposition 3, erode the equilibrium share—but along a smooth, reversible curve, not toward a cliff. The observed share of approximately 62 percent corresponds to a low disruption rate; ceding the majority would require that rate to roughly treble. The erosion that fiscal and institutional deterioration induce is gradual and, by Proposition 2, self-arresting: as the incumbent's share declines, the relief of congestion restores its relative fitness, opposing further decline. The implication is not that deterioration is harmless but that its effect is bounded and reversible, registering as a drift in the equilibrium share rather than as an impending collapse.

This is consistent with the central result of a companion study (Schmid (2026)), which synthesizes the He, Krishnamurthy, and Milbradt (2019) coordination game, the Farhi and Maggiori (2018) Cournot oligopoly framework, and the Blanchard (2019) fiscal arithmetic within a dynamic simulation. That study finds that fiscal deterioration alone is insufficient to displace the incumbent in a world without credible alternatives—the “there is no alternative” effect. The present model expresses the same insight through the structure of the successor: erosion proceeds only insofar as the diversified basket can absorb the displaced demand, and the migration is paced by the alternatives’ capacity rather than by the incumbent’s fiscal position alone.

9.2 The Self-Limiting Character of Erosion

The feedback loop analyzed in Section 6 runs in the stabilizing direction. An initial loss of market share—whether occasioned by a fiscal episode, a sanctions action, or a domestic political shock—relieves the congestion borne by the incumbent, which raises its fitness relative to the population mean (Proposition 2), which generates a restoring force opposing further loss. The dominant configuration is held in place not by an unassailable advantage but by the congestion that caps it. This is the inverse of the self-undermining dynamic posited by the Brunnermeier, Merkel, and Sanikov (2024) Safe Asset Tautology: a partial deflation of the position is not self-amplifying but self-correcting, which is why the historical record shows reversals rather than the monotone collapse a positive feedback loop would imply.

9.3 The Nature of the Successor

Proposition 4 yields a prediction concerning the successor. Under elevated institutional disruption, the successor is predicted to be not another single sovereign—which would carry its own congestion cost as reliance concentrated upon it—but a diversified, structurally robust instrument. This is consistent with the ESBies proposal of Brunnermeier, Langfield, Pagano, Reis, Van Nieuwerburgh, and Vayanos (2017) and with the observation that no single sovereign currently combines the institutional quality, fiscal credibility, capital account openness, and market depth requisite for unilateral succession. The three-asset analysis sharpens the prediction: the diversified basket captures the bulk of displaced demand, offering yield and liquidity, while gold takes the second share, consistent with its role as the institution-independent store of value. The succession is reached by gradual reallocation, and—because the curve is reversible—is not irrevocable. The comparison with a single sovereign rests on the congestion reasoning rather than on a simulated contest: the three-asset model contains no competing single sovereign as a distinct type, and the prediction follows from the high congestion cost such concentrated reliance would carry. A four-type extension that introduces a single-sovereign challenger and lets it compete with the basket directly is left to future work.

9.4 Policy Levers in Light of Propositions 2 and 3

Proposition 3 maps onto policy levers through action-consequence relationships rather than prescriptions. Strengthening intrinsic fitness a_1 —through fiscal consolidation that lowers the projected debt path, and through institutional reinforcement that preserves central bank independence and the impartiality of the legal arrangements governing U.S. debt—raises the equilibrium share (Proposition 3(ii)). Preserving network depth b_1 , by maintaining the breadth and liquidity

of the Treasury market, has the same directional effect (Proposition 3(ii)). Limiting the frequency and severity of discrete disruptions—debt-ceiling brinkmanship, partisan shutdowns, ad hoc policy reversals—reduces the realized disruption rate μ_1 and, by Proposition 3(i), slows the erosion of share.

A lever distinctive to the congestion formulation operates on the congestion cost c_1 itself. The coefficient c_1 measures the price response that mass liquidation of the incumbent would provoke, an inverse measure of the absorption capacity supplied by the holder base and the intermediation channels. Fostering structural diversity in that holder base—across official and private holders, across jurisdictions, and across the intermediation channels that absorb flight into and out of the asset—lowers the effective congestion cost and, by Proposition 3(iii), raises the equilibrium share. The Minority Principle gives this lever its content: safety is preserved not by maximizing reliance but by ensuring that reliance is distributed, so that no single channel is overwhelmed when the asset is called upon under stress. Expanding the intermediation capacity that the repo evidence shows to be finite—through central clearing capacity and balance-sheet provision that the dealer system can deploy in stress—acts on the same coefficient. This lever has no analogue in the network-effect account, in which concentration is unambiguously stabilizing; under congestion, concentration beyond the equilibrium is self-defeating, and diversity of reliance is itself a source of stability. The asymmetry of Section 4.5 sharpens the lever’s role. A disruption that erodes the share is self-correcting through the restoring force of Proposition 2; a structural rise in c_1 is not, and drives the superiority σ through unity as the coefficient rises. The diversity of the holder base that lowers c_1 therefore acts on the one channel of erosion that the restoring force leaves unopposed.

9.5 The Historical Parallel

The model’s dynamics are consistent with the historical pattern documented by Schmelzing (2020) and Chen, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2025). The British transition from safe asset hegemon to conventional sovereign borrower—which Eichengreen and Flandreau (2009) and Chitu, Eichengreen, and Mehl (2014) show began earlier than conventionally assumed, with the dollar overtaking sterling already in the mid-1920s—unfolded as a gradual migration over decades rather than as a single discontinuous break, punctuated by the large disruptions of two wars and the intervening fiscal strain. This is the pattern the smooth, disruption-driven migration of the present model generates: prolonged stable dominance, gradual erosion as the disruption rate rose, and accelerated reallocation when disruptions were large, without an endogenous tipping point. The model generates a discriminating, testable prediction for this episode: because it predicts the absence of critical slowing down, the rising variance and autocorrelation in government bond spreads that would precede a genuine bifurcation should not be found in British debt markets before the transition. A search of the long-span yield data that fails to find such precursors would favor the congestion mechanism over a tipping-point account.

10 Conclusion

This study has developed a formal dynamical model of safe asset status by transposing the replicator-mutator equation from evolutionary biology into the framework of financial economics. The study has drawn on the modern frameworks of Gorton and Pennacchi (1990), Woodford (1990), Holmström and Tirole (1998), Bansal and Coleman (1996), Krishnamurthy and Vissing-Jorgensen (2012), Brunnermeier, Merkel, and Sannikov (2024), and He, Krishnamurthy, and

Milbradt (2019) from financial economics. The evolutionary framework—drawing on Eigen (1971), Taylor and Jonker (1978), Hofbauer and Sigmund (1998), and Wilke, Wang, Ofria, Lenski, and Adami (2001)—has been shown to unify these insights within a single dynamical system.

The model’s principal contribution, formalized as Proposition 2, is the identification of a self-limiting feedback loop connecting market share, congestion, and relative fitness. In the standard Eigen model the superiority parameter is fixed and a master sequence is maintained or lost through an error catastrophe; in a network-effect model the superiority falls as share falls, so erosion compounds. In the congestion model developed here the loop runs in the stabilizing direction: as the incumbent’s share erodes from its majority, the relief of congestion raises its relative fitness, generating a restoring force that opposes further erosion. This is the safe asset paradox stated as a stabilizing principle—the Minority Principle—and it supplies the restoring force the empirical literature has invoked, without formalizing, to account for the partial recoveries in the dollar’s reserve share. The transition between a Treasury-dominant and a basket-dominant configuration is smooth, reversible, and free of an endogenous tipping point; the model further predicts the absence of critical slowing down, a signature that discriminates it from a catastrophe mechanism.

The convergence of this framework with the game-theoretic approach developed in a companion paper (Schmid (2026))—which models safe asset status as the outcome of three interconnected games (investor coordination, Cournot oligopoly, and fiscal sustainability) with four endogenous feedback channels (the Triffin dilemma, Gopinath and Stein (2021) invoicing complementarities, shortage-responsive convenience yields, and Nurkse instability)—strengthens confidence in the shared central finding: institutional erosion must accompany fiscal deterioration for status loss to proceed, and even then the loss is gradual and reversible rather than catastrophic. The present paper’s calibration places the system at a congestion-stabilized majority equilibrium of approximately 62 percent, inside the range in which the restoring force operates.

Three caveats are warranted. First, the model is deliberately stylized; the fitness functions are parametric, the mutation rates are treated as exogenous, and the mapping from biological parameters to financial observables requires the exercise of judgment. Second, the model does not incorporate the strategic behavior of large institutional players—central banks, sovereign wealth funds—whose portfolio decisions may influence the equilibrium directly. The incorporation of heterogeneous, strategically interacting agents constitutes an extension, alongside the state-dependent fitness specification of equation (5) that would capture the flight-to-safety mechanism the present stationary specification omits. Third, while the model identifies which levers operate on the equilibrium share (Section 9.4), it does not endogenize the political economy of whether those levers will be exercised. The comparative statics of Proposition 3 indicate that strengthening fundamentals, preserving market depth, fostering diversity in the holder base, and limiting discrete disruptions each raise the equilibrium share; whether these levers are exercised is a question the model does not address.

Subject to these caveats, the model offers a disciplined framework for analyzing the dynamics of safe asset status—one that integrates insights from information economics, macroeconomic theory, game theory, and evolutionary biology into a single, formally coherent structure. The historical record documented by Schmelzing (2020) and Chen, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2025)—encompassing approximately four transitions over seven centuries, in none of which has a sovereign that forfeited safe asset provider status subsequently regained it—is sometimes read as evidence of irreversibility. The present model does not rest on that reading. Its equilibrium curve is reversible, so the absence of historical recoveries is attributed not to an endogenous trap but to the persistence of the underlying disruptions that raised the incumbent’s disruption rate and to

the congestion-stabilization that a successor acquires once it holds the majority. Recovery is difficult not because erosion is self-amplifying but because the conditions that drove it—sustained fiscal and institutional deterioration—rarely reverse, and because the safety that migrates to a diversified successor is, by the same Minority Principle, stabilized in its turn.

A Proofs of the Main Propositions

Proof of Proposition 1 (Existence and Stability)

Each fitness function $f_i(x) = a_i + b_i x^\gamma - c_i x^\eta$ with $0 < \gamma < 1 < \eta$ is single-peaked on $(0, 1)$. Its derivative $f'_i(x) = \gamma b_i x^{\gamma-1} - \eta c_i x^{\eta-1}$ is strictly positive as $x \rightarrow 0^+$ (the term $x^{\gamma-1}$ diverges while $x^{\eta-1} \rightarrow 0$) and, for the calibrated parameters, satisfies $f'_i(1) = \gamma b_i - \eta c_i < 0$; by continuity f_i attains an interior maximum at the unique $\hat{x}_i$ solving $\gamma b_i \hat{x}_i^{\gamma-1} = \eta c_i \hat{x}_i^{\eta-1}$, rising for $x < \hat{x}_i$ and falling for $x > \hat{x}_i$.

The replicator-mutator map carries the simplex Δ^2 continuously into itself, so by the Brouwer fixed-point theorem a rest point exists. An interior rest point satisfies the stationarity conditions $\dot{x}_i = 0$. The dominant share's condition, after isolating the faithful-replication term, reads

$$(f_1 - \phi) x_1^* = \mu_1 f_1 x_1^* - \sum_{j \neq 1} \mu_j \tau_{1j} f_j x_j^*,$$

so $\sigma = f_1/\phi$ exceeds unity exactly where the mutation outflow from the incumbent exceeds the mutation inflow to it. At the calibrated parameters this holds for μ_1 between about 0.010 and 0.217, a range that contains the calibrated rate; below it the inflow, which does not vanish with μ_1 , exceeds the outflow, and $\sigma < 1$. Computed along the dominant branch, $f_2(x_2^*) > f_1(x_1^*)$ holds for μ_1 up to about 0.021. Both relations are confirmed at the calibrated equilibrium $x^* = (0.620, 0.270, 0.110)$, where $f = (1.222, 1.223, 1.117)$, $\phi = 1.211$, and $\sigma = 1.009$.

Because $x_1^* > \hat{x}_1$, the dominant equilibrium lies on the falling branch of f_1 , where $f'_1(x_1^*) < 0$ (numerically -0.851). Local asymptotic stability is established from the Jacobian of the reduced two-dimensional system obtained by eliminating $x_3 = 1 - x_1 - x_2$. Its eigenvalues at x^* are -0.2053 and -0.0552 , both with strictly negative real part, and along the dominant branch both remain negative at every μ_1 below the majority threshold. At the calibrated rates, numerical integration of the flow from initial conditions distributed across Δ^2 converges to x^* in every case, so the equilibrium is globally attracting on the interior. $\square$

Proof of Proposition 2 (Self-Limiting Erosion)

Along the equilibrium manifold $x^*(\mu_1)$, differentiate $\sigma = f_1(x_1^*)/\phi(x^*)$ with respect to μ_1 :

$$\frac{\partial \sigma}{\partial \mu_1} = \frac{1}{\phi} f'_1(x_1^*) \frac{\partial x_1^*}{\partial \mu_1} - \frac{f_1}{\phi^2} \frac{\partial \phi}{\partial \mu_1}.$$

On the dominant branch $f'_1(x_1^*) < 0$ (Proposition 1), and $\partial x_1^*/\partial \mu_1 < 0$ (Proposition 3), so the first term is strictly positive: as the incumbent sheds share, the relief of congestion ($-c_1 x_1^\eta$ falling in magnitude) outweighs the loss of network benefit ($b_1 x_1^\gamma$ declining) on this branch, and $f_1(x_1^*)$ rises. The displaced share migrates to the alternatives, whose fitness contributions to ϕ rise modestly; direct computation at the calibrated parameters gives a net $\partial \sigma / \partial \mu_1 = +0.890 > 0$ at the baseline, and σ remains increasing past the loss of the majority at $\mu_1 \approx 0.055$, to its maximum near $\mu_1 = 0.112$, where $\sigma = 1.074$. The positive sign establishes the restoring force: a disruption that erodes x_1^* raises the incumbent's relative fitness and thereby opposes the erosion. $\square$

Proof of Proposition 3 (Comparative Statics)

At a stable interior equilibrium the equilibrium condition $F(x^*, \theta) = 0$ holds for each parameter θ . The implicit function theorem gives $dx^*/d\theta = -[J(x^*)]^{-1}(\partial F/\partial\theta)$, where the Jacobian $J(x^*)$ is non-singular because its eigenvalues have strictly negative real parts (Proposition 1). Evaluating the required derivatives at the calibrated equilibrium yields $\partial x_1^*/\partial\mu_1 = -3.24$, $\partial x_2^*/\partial\mu_1 = +0.70$, and $\partial x_3^*/\partial\mu_1 = +2.54$, establishing (i); the signs of $\partial x_1^*/\partial a_1 > 0$, $\partial x_1^*/\partial b_1 > 0$, and $\partial x_1^*/\partial c_1 < 0$ follow from the corresponding entries of $\partial F/\partial\theta$ and establish (ii)–(iii); (iv) is Proposition 2. For (v), note that $J(x^*)$ remains non-singular along the entire dominant branch—no eigenvalue crosses zero as μ_1 varies—so the equilibrium is a continuously differentiable function of μ_1 by the implicit function theorem, with no fold (saddle-node) point on that branch. For μ_1 above about 0.0074, solving the dynamics from initial conditions across the simplex at each μ_1 yields the identical limit, confirming the absence of bistability and hence of hysteresis there; below that rate a second limit exists, on a branch that ends in a fold at $\mu_1 \approx 0.0074$ (Section 5.5). $\square$

Proof of Proposition 4 (Survival of the Flattest)

Along the dominant branch above the threshold of Section 5.5, x_1^* is strictly decreasing in μ_1 and x_3^* strictly increasing, by computation of the branch; x_2^* is strictly increasing for μ_1 above about 0.012 and below it varies between 0.267 and 0.268. It remains to establish the ordering among the alternatives and the eventual plurality of the basket. Consider the parity plane $x_2 = x_3 = y$. The difference in the two velocities there is

$$\dot{x}_2 - \dot{x}_3 = \mu_1(\tau_{21} - \tau_{31}) f_1(x_1) x_1 + y [C_2 f_2(y) - C_3 f_3(y)],$$

with retention constants C_2, C_3 near unity. The first term is strictly positive because $\tau_{21} = 0.55 > \tau_{31} = 0.45$: demand fleeing the incumbent favors the basket over gold. The second term is negative only for y below about 0.13, where gold's larger intrinsic fitness ($a_3 > a_2$) outweighs the basket's larger network benefit ($b_2 = 0.778 > b_3 = 0.663$) and smaller congestion cost ($c_2 < c_3$); evaluated over the whole parity plane, the sum is positive for every μ_1 above about 0.014 (the calibrated rate lies in this range). For those rates $\dot{x}_2 - \dot{x}_3 > 0$ on the parity plane, so the region $\{x_2 > x_3\}$ is positively invariant and $x_2^* > x_3^*$; between the threshold of Section 5.5 and $\mu_1 \approx 0.014$ the ordering holds on the computed branch, where x_2^* stands near 0.27 and x_3^* below 0.10. Combined with $x_1^* \downarrow$ and $x_2^* \uparrow$, the incumbent's and basket's shares cross at a finite mutation rate (numerically near $\mu_1 = 0.085$), beyond which $x_2^* > \max(x_1^*, x_3^*)$; at $\mu_1 = 0.30$ the equilibrium is $x^* = (0.05, 0.59, 0.36)$. The crossing is continuous and involves no collision of equilibria, so no saddle-node bifurcation occurs: the succession of the basket is reached by gradual reallocation. $\square$

Proof of Proposition 5 (Channel-Specificity of the Restoring Force)

Along the equilibrium manifold the superiority is $\sigma = f_1(x_1^*)/\phi(x^*)$. Differentiating with respect to c_1 ,

$$\frac{\partial \sigma}{\partial c_1} = \frac{1}{\phi} \left[\frac{\partial f_1}{\partial c_1} + f_1'(x_1^*) \frac{\partial x_1^*}{\partial c_1} \right] - \frac{f_1}{\phi^2} \frac{\partial \phi}{\partial c_1}, \quad \frac{\partial f_1}{\partial c_1} = -(x_1^*)^\eta < 0.$$

The direct term is strictly negative. On the dominant branch $f_1'(x_1^*) < 0$ (Proposition 1) and $\partial x_1^*/\partial c_1 < 0$ (Proposition 3(iii)), so the indirect term $f_1'(x_1^*) \partial x_1^*/\partial c_1$ is positive: the share relinquished as c_1 rises relieves congestion and partially restores f_1 . The mean-fitness term is of smaller

magnitude, the displaced share accruing to assets whose contributions to ϕ change by an offsetting but smaller amount. Evaluating at the calibrated equilibrium gives $\partial\sigma/\partial c_1 = -0.022 < 0$, the direct penalty dominating; the sign is opposite to $\partial\sigma/\partial\mu_1 = +0.890$ of Proposition 2. Solving the equilibrium across c_1 confirms that σ decreases monotonically and crosses unity near $c_1 = 1.75$ ($x_1^* \approx 0.48$), while at the calibrated disruption rate the equilibrium remains unique throughout, so no bifurcation accompanies the decline: the incumbent cedes the majority at $c_1 \approx 1.66$ and the basket overtakes it at $c_1 \approx 2.09$, each reached by continuous migration. $\square$

B Baseline Parameter Values

Table B.1 reports the baseline parameter values employed in all numerical simulations unless otherwise indicated.

Table B.1. Baseline Calibration of the Replicator-Mutator Safe Asset Model

Parameter	Asset 1 (UST)	Asset 2 (Basket)	Asset 3 (Gold)	Interpretation
a (intrinsic safety)	0.935	0.940	0.960	Freedom from default and institutional risk
b (network benefit)	1.048	0.778	0.663	Liquidity and convenience premium
c (congestion cost)	1.300	0.980	1.600	Price response to mass liquidation
γ (benefit exp.)	0.60	0.60	0.60	Diminishing returns to depth
η (congestion exp.)	2.00	2.00	2.00	Convex crowding cost
μ (mutation rate)	0.02	0.04	0.03	Institutional disruption

Calibration target (Section 8.1): equilibrium shares $x^ = (0.620, 0.270, 0.110)$, matching the U.S. Treasury share of the global safe asset universe (marketable sovereign debt rated Aa3/AA– and above, plus monetary gold; defensible range 0.59–0.72). Resulting superiority $\sigma(x^*) = 1.009$.*

Mutation target matrix τ (off-diagonal elements; rows denote destination i , columns denote source j):

	From UST ($j = 1$)	From Basket ($j = 2$)	From Gold ($j = 3$)
To UST ($i = 1$)	–	0.46	0.60
To Basket ($i = 2$)	0.55	–	0.40
To Gold ($i = 3$)	0.45	0.54	–

Mutation targets reflect substitution patterns in reserve management: demand fleeing the incumbent favors the diversified basket over gold ($\tau_{21} > \tau_{31}$). Each column sums to one.

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