

The Kelly Criterion and Learning in Competitive Insurance Underwriting

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Abstract. A rating plan gives an insurer a price for an account and no statement of what the price is for. This paper gives underwriting an objective, the long-run growth of the insurer's surplus, which the Kelly criterion states as the maximization of the expected mean geometric rate of growth, and asks what an insurer must know to pursue it and how much of that it can learn from its own operations. The criterion names four things: the account's expected loss, the variability of that loss against the insurer's surplus, the value of keeping the account, and the chance that a quote wins it. A rating plan supplies a version of the first. The insurer can learn all four from what it already holds, its claim files, the loss runs of the accounts it is asked to quote, its own retention, and the broker's report of the quotes it lost, and it can put what it learns to work in two ways: through its rating plan, with the learned estimate in place of the plan's, or through the Kelly decision, as a deviation from the plan's price. In a stylized market for middle-market general liability beside three conventional insurers, both ways reach the highest growth of surplus on the record, and the growth they reach is the same. They differ in the book each builds, in the prices each quotes, and in the path by which each arrives. Learning is what grows the surplus; the Kelly decision is one way to put it to work, and the insurer's own rating plan is another.

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Contents

Abbreviations and Acronyms	ii
Greek Letters	ii
Latin Symbols with More Than One Meaning	iii
1 Introduction	1
2 Growth as the Objective, and Learning as Its Source	4
3 A Stylized Market	6
4 What Learning Is Worth	8
5 Two Routes to the Same Growth	11
6 When the Routes Part	13
7 The Kelly Underwriter	14
8 What Follows for an Insurer, and Conclusion	16
A The Market: Its Assumptions, Its Insurers, and Its Loss Process	19
B The Rating Plans	25
C The Capital Standard	26
D The Kelly Decision and the Estimators	27
E The Kelly Underwriter's Computation, Step by Step	36
F The Registry of Predictions and Outcomes	37
G The Kelly Underwriter's Parts, and the Market around It	41
H The Sensitivity Program, Arm by Arm	46
I The Quote Data and the Further Markets	51
Bibliography	54

Abbreviations and Acronyms

ACORD	Association for Cooperative Operations Research and Development
GL	General liability
GLM	Generalized linear model
IRIS	Insurance Regulatory Information System
NAIC	National Association of Insurance Commissioners
RBC	Risk-based capital

Greek Letters

α	alpha	Permanent part of a trait (§A.6)
β	beta	Discount factor of the renewal credit: at the insurer's own growth of surplus or at the return on funds (§D.1.3; Section 4 names the two discounts)
Δ	Delta	Logarithm of the holder's quote over the lowest invited quote (§A.4)
ε	epsilon	Expenses as a share of premium (§A.3)
ζ	zeta	Latent log price of the demand model, over the price its deviation is taken from (§D.2.3)
η	eta	Yearly shock to the drifting part of a trait (§A.6)
κ	kappa	A cell's factor on the tracked expected loss (§D.2.2); the capital requirement's scale (Appendix C)
λ	lambda	An account's expected number of claims in a policy year (§A.1); $\hat{\lambda}$, the tracked claim rate (Equation D.5); λ_0 , the claims a year at the median size (§A.6)
μ_ζ	mu	Mean of the demand model's ζ (§D.2.3)
ν	nu	Mean of the permanent traits' sum, in the schedule modification (Appendix B)
ξ	xi	Standard normal draw of a case reserve's error (§A.6)
π	pi	Provision for expenses and profit in a plan's premium (§A.3); the posterior density of the demand model's parameters (Equation D.7)
ρ	rho	Persistence of the drifting part of a trait (§A.1); the capital requirement's reserve-charge ratio (Appendix C); with subscript f or s , the fitted persistence of an autocovariance (§D.3)
σ	sigma	Spread of the demand model's ζ (§D.2.3); with subscript c , of a case reserve's error (§A.6)
τ	tau	Mean time from booking to payment, years (§D.1.2)
Σ	Sigma	Covariance of a file's observations (§D.3); the summation sign (Equation D.3, Appendix B)
ϕ	phi	Standard normal density (§D.3)
Φ	Phi	Standard normal distribution function (§D.2.3)
ψ	psi	Complement of credibility in the loss-ratio method's indication (§A.3)
ω	omega	Weight of a quote in the demand model's likelihood (§D.3)

Latin Symbols with More Than One Meaning

The letters E, C, L, and T also name the four further markets of Section 6, run in Appendix I.

- A* An account's capped losses on the file it is rated on (§A.3); a policy's capped reported losses, in the GLM insurer's fit (Appendix B); with subscript f or s , a fitted constant of an autocovariance (§D.3)
- a* Reported losses, in the loss-ratio method's indication (Equation A.2) and of a cell, a_c (Equation D.6); the loss adjustment expense's share (Equation D.5)
- B* With subscript c , a class's expected capped loss of a year per unit of size (Appendix B); with subscript f or s , a fitted coefficient of an autocovariance (§D.3)
- c* An account's class (§A.3); as a subscript, a cell (§D.2.2) or a case reserve (σ_c); c_x , the return on the premium held (§D.1.2); c_f and c_s , covariances of the observations (§D.3)
- d* Base of the weights of the experience modification (§A.3); with subscript it , the drifting part of a trait (§A.6)
- E* A class's expected capped losses, E_{rep} their expected reported part (§A.3); expected reported losses, in the GLM insurer's fit (Appendix B); $\mathbb{E}$, expectation (§D.1.1)
- f* The indicated change of level (Equation A.2); the GLM insurer's change of level or a fitted factor (Appendix B)
- g* An insurer's growth (Equation A.3); $\hat{g}$, its mean yearly log growth of surplus over the credit's window (§D.1.3)
- j* The age of a year, in the experience modification (§A.3); a quote, in the demand model's likelihood (§D.3)
- k* The buyer's cost of shopping, k' its cost of switching (§A.4); years ahead (§D.1.3); a lag (§D.3)
- m* With subscript 1 or s , a margin per unit of net premium (§D.1.3); m_0 , the mean log claim on the bureau's curve (§D.3)
- n* Premium net of expenses (§D.1.2); reported claims, in the loss-ratio method's indication (Equation A.2) and in the estimator's observations, n_s^o the open claims among them (§D.3)
- s* The scale of the buyer's response (§A.4); as a subscript, later renewals (§D.1.3), the severity trait, or an account-year (§D.3); s_1^2 and s_2^2 , variances in the schedule modification (Appendix B)
- t* A year (§D.1.3, §A.6); the t statistic of a contrast (§A.5)
- u* The occurrence limit (Equation D.5); with subscript it , the frequency trait (§A.6)
- v* $\hat{v}$, the estimated variance of an account's loss (§D.1.2); v_0 , the variance of a log claim on the bureau's curve (§D.3)
- w* With subscript j , the weight of a year in the experience modification (Appendix B); a file's length in years (§D.3)
- x* $\bar{x}$, the plan's expected loss (Equation A.1); $\hat{x}$, the estimated expected loss, $\hat{x}_0$ before a cell's factor (§D.1.2, §D.2.1)
- Y* Claim size at the tracked scale (Equation D.5); $\bar{y}$, a mean log claim size (§D.3)
- Z* Classical credibility (Equation A.2, Appendix B), of a cell Z_c (Equation D.6); Z_{exp} , the experience modification's credibility (§A.3)

1 Introduction

An insurer that competes for an account decides two things at once: at what price to quote, and whether it wants the account at the price the market will bear. For neither does it know what it needs. The first requires the account's loss distribution, which no one observes; the second requires the chance of winning at each price, which depends on what competitors quote and on how the buyer chooses. The industry's answer is the rating plan: a class loss cost, an experience modification, a schedule modification, and a provision for expenses and profit, with the insurer's rate level moved once a year by what its book has earned against what it was priced to earn.¹ The plan states a price. It does not state what the price is for.

This paper gives underwriting an objective and asks what an insurer needs in order to pursue it. The objective is the long-run growth of surplus. The Kelly criterion states it: an insurer that maximizes the expected logarithm of its surplus at the end of a horizon maximizes the expected mean geometric rate at which the surplus grows, and in repeated independent bets on known odds no other rule grows wealth faster in the long run.² Applied to one account, the criterion gives a price from four things: the account's expected loss; the variability of that loss against the insurer's surplus, which is the criterion's weight on risk; the value of keeping the account in the years after this one; and the chance that the quote wins. A rating plan supplies a version of the first and none of the other three.

An insurer can learn all four from what it already holds. Its claim files record every claim of every account it has written; the loss runs of the accounts it is asked to quote record their claims elsewhere; its own retention records which accounts stayed at which price; and the broker reports the price at which a lost quote lost. From these the insurer tracks each account's claim frequency and claim size as the files lengthen, corrects for the fact that the accounts it won are not a random sample of the accounts it quoted, and fits the chance that a quote holds an account at a given price. Learning in this sense is the source of what the objective needs. The insurer can put what it learns to work in two ways. It can price through its rating plan, with the learned estimate of each account's loss in place of the plan's: *learning at the plan's pricing*, the plan route. Or it can price by the Kelly decision, as a deviation from the plan's price: *learning at the Kelly decision*, the Kelly route. The two routes price the same learned estimate and differ in the rule that turns it into a quote.

We measure the two in a stylized market for middle-market general liability (GL). Three conventional insurers compete in it, each named by how it sets its price: a *Bureau insurer*, which prices on the bureau's plan and refiles its own multiplier when its experience departs from the plan; a *Loss-ratio insurer*, which moves its rate level each year by the loss-ratio method; and a *GLM insurer*, which fits a generalized linear model (GLM) to its own book and adds its relativities by class and size to the plan. A fourth seat is filled in turn by the insurers under study, the three held as they are. The market has 4,000 accounts and runs for 28 record years; it is run 200 times from 200 draws of its random streams, the same draws for every filling of the fourth seat, so that a comparison between two fillings is a comparison of the same market under two insurers. Growth is the surplus at the close of the record years over the endowment, net of the return on funds, stated as a rate of

¹Geoff Werner and Claudine Modlin, *Basic Ratemaking*, 5th ed. ([Arlington, VA]: Casualty Actuarial Society, 2016), chaps. 8, 12, and 15; Ginda Kaplan Fisher, Lawrence McTaggart, Jill Petker, and Rebecca Pettingell, *Individual Risk Rating* (Casualty Actuarial Society study note, 2019).

²John L. Kelly Jr., "A New Interpretation of Information Rate," *Bell System Technical Journal* 35, no. 4 (1956): 917–926; Leo Breiman, "Optimal Gambling Systems for Favorable Games," in *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, vol. 1, ed. Jerzy Neyman (Berkeley: University of California Press, 1961), 65–78.

growth a year; a difference between two fillings is stated as a percentage of terminal surplus, and the record resolves a difference whose t statistic exceeds 2. The market is stylized, and no observed market has an insurer that prices by the Kelly decision; the inputs follow industry practice where practice is documented, and each is listed with its grounding (Appendix A).

Three findings are on the record. First, learning is what grows surplus. A fourth conventional insurer, a second of the Loss-ratio type, grows at 1.93 percent a year, 1.71 times its endowment over the record years. The plan route grows at 3.39 percent a year, 2.55 times its endowment: learning each account's loss distribution and pricing it through the plan adds 49.2 percent to terminal surplus. The Kelly decision applied to the plan's own estimate adds nothing, because a pricing rule is worth what the information behind it is worth, and the plan's estimate carries no more information when a different rule prices it. Second, the two routes reach the same growth. The Kelly route grows at 3.43 percent a year, 2.57 times its endowment, and its difference from the plan route, 1.1 percent of terminal surplus, lies within a margin of equivalence of 0.10 percentage points of growth a year, set before the test was run. It does not take the Kelly decision to realize the gain from learning. Third, the two routes reach one destination by different books and different paths. The Kelly route quotes renewals 12.6 percent above the plan's price and new business 8.4 percent below it; it holds 49 percent of the market's accounts at the close of the record years where the plan route holds 67 percent; accounts change insurer around it at 26 percent a year against 16; and it trails the plan route through the first two decades, through record year 23 by a difference the record resolves (one whose t statistic exceeds 2; Section 3 states the test), and converges with it only over the final years, its growth steadier along any one path and more dispersed across paths. A management that reads growth at year ten or year twenty sees a route that is losing.

Whether the two routes stay together is a property of the market. Where the filed rate band, inside which an insurer may move a price, is 40 percent either way in place of 25, the Kelly route trails the plan route by 14.2 percent of terminal surplus; a buyer more responsive to price moves it behind by 3.4 percent, and a buyer less responsive moves it ahead by 3.1. In a smaller insurer it leads by 2.3 percent, in a market with a tenth of the accounts by 12.3, and under a rising severity trend that no insurer in the market projects by 7.2; beside a second insurer that learns, the two routes stay within the margin of each other. A third insurer under study, *the Kelly underwriter*, bundles the learning and the decision in one architecture. It grows at 2.50 percent a year, 2.00 times its endowment, 42.4, 53.7, and 38.2 percent more terminal surplus than the three conventional insurers, and, with the anchored credit, 17.3 percent less than the plan route; its leads over the conventional insurers hold across 79 variations of the market but one, and under the rising severity trend it trails all three. The one part of its architecture it cannot do without is the value of keeping an account: priced without it, the Kelly underwriter falls behind all three conventional insurers.

For an insurer the findings read as follows. The return is in the learning, and the rating plan can carry it: an insurer that estimates each account's loss distribution from its own files and prices through the plan it already has reaches the highest growth on the record, shared with the Kelly route within the margin, without changing its pricing architecture. The Kelly decision is a choice about the book the insurer builds and the path it travels, not about the growth it reaches at the record's calibration: a smaller book with a higher renewal margin, lower new-business prices, more turnover, and two decades of trailing before it converges. Where the market departs from the record's, in a smaller insurer, in fewer accounts, or under a rising severity trend, the decision adds to growth; where the rate band is wide or the buyer responsive to price, it costs. And the form in which the value of keeping an account enters the price decides whether the decision adds or costs:

anchored at the plan's price, the decision reaches the plan route's growth; in proportion to the price quoted, it rewards a markup with a value the markup does not buy and falls short of it (Section 2).

Five limits bound the findings, each stated here once and in full in §8.2. The market is one line of business with independent accounts, no loss trend except in the one market run with one, no underwriting cycle, no reinsurance, and conventional insurers that do not respond to a learner beyond their own rules; its buyer responds to price alone. The two routes are run at the record's calibration of the rate band and the buyer's responsiveness and at one other value of the band and two of the responsiveness. The criterion's weight on risk, the variance term, is small at this market's surplus, so the record tests the criterion's rule for pricing under uncertainty and not its weighting of risk. The horizon is known to the insurer, and growth is read at the horizon. And every difference is measured within one model over 200 replications; its t statistic states the precision of the simulation, not of an observed market.

The paper draws on three literatures. The first is the Kelly criterion of Kelly and Breiman, whose second-order form is a mean–variance rule with the weight on variance fixed by the criterion.³ An insurer does not know the odds, and the paper's findings are statements about that case. The second is the theory of insurance pricing under asymmetric learning, in which the insurer that holds an account learns about it what competitors do not, and without commitment the equilibrium has a low first price, the “lowballing” of that literature, recovered through the margins of later renewals on the accounts that stay.⁴ In our market the holder reads its own claim file where a competitor reads a broker's loss run, every contract is annual, and the insurers that value retention arrive at new-business prices below their renewal prices without a model of that equilibrium (§5.2). The empirical counterpart is the measured difference between new and renewal business: in each of the 25 books of business that Wu and Lin examine the loss ratio of new business is above that of renewals.⁵ The third is the study of pricing algorithms in competition, which asks whether algorithms that learn, or that respond to rivals' prices, raise prices.⁶ That question is about the level of prices among sellers whose algorithms meet one another; the question here is what an objective and the learning are worth to one insurer among insurers that do not learn as it does.

Section 2 states the approach: the objective, the four things the decision needs and how each enters the price, and what learning supplies. Section 3 describes the market. Section 4 reports what learning is worth, Section 5 the two routes and how they differ, Section 6 the markets in which they part, and Section 7 the Kelly underwriter. Section 8 states what follows for an insurer, the limits,

³Kelly, “Interpretation of Information Rate”; Breiman, “Optimal Gambling Systems”; Edward O. Thorp, “The Kelly Criterion in Blackjack Sports Betting, and the Stock Market,” in *Handbook of Asset and Liability Management*, vol. 1, ed. Stavros A. Zenios and William T. Ziemba (Amsterdam: North-Holland, 2006), 385–428.

⁴Howard Kunreuther and Mark Pauly, “Market Equilibrium with Private Knowledge: An Insurance Example,” *Journal of Public Economics* 26, no. 3 (1985): 269–288; Tore Nilssen, “Consumer Lock-in with Asymmetric Information,” *International Journal of Industrial Organization* 18, no. 4 (2000): 641–666; Thomas de Garidel-Thoron, “Welfare-Improving Asymmetric Information in Dynamic Insurance Markets,” *Journal of Political Economy* 113, no. 1 (2005): 121–150; Ruo Jia and Zenan Wu, “Insurer Commitment and Dynamic Pricing Pattern,” *Geneva Risk and Insurance Review* 44, no. 1 (2019): 87–135; and, for evidence consistent with the theory, Stephen P. D’Arcy and Neil A. Doherty, “Adverse Selection, Private Information, and Lowballing in Insurance Markets,” *Journal of Business* 63, no. 2 (1990): 145–164.

⁵Cheng-Sheng Peter Wu and Hua Lin, “Large Scale Analysis of Persistency and Renewal Discounts for Property and Casualty Insurance,” *Casualty Actuarial Society E-Forum* (Winter 2009): 398; Stephen P. D’Arcy and Neil A. Doherty, “The Aging Phenomenon and Insurance Prices,” *Proceedings of the Casualty Actuarial Society* 76 (1989): 24–44.

⁶Emilio Calvano, Giacomo Calzolari, Vincenzo Denicolò, and Sergio Pastorello, “Artificial Intelligence, Algorithmic Pricing, and Collusion,” *American Economic Review* 110, no. 10 (2020): 3267–3297; Stephanie Assad, Robert Clark, Daniel Ershov, and Lei Xu, “Algorithmic Pricing and Competition: Empirical Evidence from the German Retail Gasoline Market,” *Journal of Political Economy* 132, no. 3 (2024): 723–771; Zach Y. Brown and Alexander MacKay, “Competition in Pricing Algorithms,” *American Economic Journal: Microeconomics* 15, no. 2 (2023): 109–156.

and the conclusion. The appendices carry the market's assumptions and its construction in full, the rating plans, the capital standard, the Kelly decision and the estimators with their equations, the Kelly underwriter's computation step by step, the registry of predictions and outcomes, the Kelly underwriter's parts and the market around it, the sensitivity program, and the quote data.

2 Growth as the Objective, and Learning as Its Source

This section states the approach in the terms an underwriter uses: what the Kelly criterion asks of a price and why an insurer would adopt it, the four things the decision needs and what each means, how the value of keeping an account enters the price, what learning supplies and from what data, and the two ways to use what is learned. Appendix D states the same with its equations and the estimators in full.

2.1 The Objective, and the Decision It Gives

An insurer's surplus compounds. What it writes this year changes the surplus from which it writes next year, and over a horizon of many years the surplus that results is the product of the year-by-year growth factors. The Kelly criterion takes the long-run growth rate of surplus as the objective: the insurer chooses its prices so as to maximize the expected logarithm of its surplus at the horizon, which is the same as maximizing the expected mean geometric rate at which the surplus grows from year to year. The criterion's property, for a better who knows the odds, is that no other rule grows wealth faster in the long run. An insurer does not know the odds, and the paper's record is a measurement of what the objective is worth to an insurer that has to estimate them.

For one account the objective reduces to a rule. Writing an account at a premium changes the surplus by the premium net of expenses less the account's loss. For an account small against the surplus, the expected change in the logarithm of surplus is the expected margin per unit of surplus less half the variance of the loss per unit of surplus squared: a mean–variance rule, with the weight on the variance set by the objective and not by the insurer. Two further terms enter in a market where the account can be kept. The premium is held until the claims are paid and earns the return on funds in the meantime. And an account written today is likely to be held in the years to come, so the margins of those years count, discounted at the insurer's own rate of growth, since a margin earned later is earned against a larger surplus. With n the premium net of expenses, $\bar{n}$ the same at the price the plan would charge, $\hat{x}$ and $\hat{v}$ the account's estimated expected loss and its variance, S the surplus, c_x the return the premium earns until the loss is paid, for which the loss is netted, and V the value of keeping the account per unit of $\bar{n}$, the expected increment to the logarithm of surplus is

$$G(P) = \frac{n - \hat{x}(1 - c_x) + \bar{n}V}{S} - \frac{\hat{v}}{2S^2}, \quad (1)$$

and the *Kelly decision* is the premium that maximizes the chance of winning the account at that premium times the increment it would earn. The decision needs four things of an account, and each has a meaning in underwriting terms. The expected loss $\hat{x}$ is what a rating plan also estimates. The variance $\hat{v}$ is the criterion's weight on risk: of two accounts with one expected loss, the one whose loss is more variable is charged more, and an insurer with less surplus charges more for the same variability. The value of keeping the account, V , is the margin of each later renewal times the chance of reaching it, discounted, summed over the years to the horizon; the paper calls it the *renewal credit*. And the chance of winning at each premium is the insurer's own hold curve, fitted

to the quotes it won and lost. The decision also sets a floor, the premium at which the increment is zero, below which the insurer does not write; the floor keeps the credit's first form, in proportion to the premium quoted (Appendix E).

How the value of keeping an account enters the price matters, and the paper's construction anchors it. The chance of winning multiplies the whole increment, so the value of keeping the account is already counted in full by the chance that the quote holds it. The renewal credit therefore enters at the price the plan would charge, the *anchored credit*: a markup today buys this year's margin and the chance of keeping the account, and it does not buy more continuation value, because next year's quote is next year's rule applied to next year's estimate, and the buyer's decision to shop compares the holder's renewal quote with the competition's. A credit that rises with the premium quoted rewards a markup with a value the markup does not buy; the record measures what each form is worth (Section 4, Appendix F).

2.2 Learning from What an Insurer Holds

A rating plan reads an account's history through one number, the experience modification on its capped losses. The learning of this paper reads the whole file and estimates each account's loss distribution from it, in three parts, each from data the insurer holds. First, the account's claim frequency relative to its class and the scale of its claim sizes are tracked from the files the insurer reads, its own claim file on an account it holds and the broker's loss run on an account it is asked to quote, by credibility weighting, with the weight on each year of the file set by how much the year's claims carry and by how far back it lies, and with the persistence of an account's departure from its class estimated from the insurer's own files rather than assumed. The expected loss and the variance follow from the tracked frequency and scale, and the variance is the quantity a plan does not hold.

Second, the insurer corrects for the business it wins. Where several insurers quote from estimates with error and the buyer favors the lowest price, an insurer wins more often where its own estimate is low, so the losses of the business won run above the estimates that won it. The insurer measures this on its own book: it sorts the policies it has written into cells, renewals and new business by size band, compares each cell's losses with the estimates that priced them, and carries a factor by cell into the price. The cells take the place of a rate level.

Third, the insurer fits its own hold curve, the chance that a quote at a given deviation from its working price holds the account, from the only information it has on competitors' prices: the broker's report of the price at which a lost quote lost, and the fact that a quote that bound was not undercut. The fit allows for the quotes that bound, whose competing prices the insurer never sees, and the decision averages the chance of winning over the uncertainty of the fit. The renewal credit's five inputs, the retention and the margin at a first renewal and at later ones and the insurer's own growth, are read from the insurer's own book over a window of years.

What is learned can be used in two ways, and the paper measures both. On the plan route the learned expected loss enters the rating plan where the plan's estimate stood, and the plan's own rules, the rate level moved by the loss-ratio method and weighted with competitors' rates, and the schedule credit on new business, turn it into a quote. On the Kelly route the same estimate is priced by the Kelly decision as a deviation from the plan route's price, inside the filed rate band of 25 percent either way, so that the Kelly route contains the plan route as the choice of a zero deviation at every account. Both routes price at the plan's rate level and without the correction by cell, which is the Kelly underwriter's. The Kelly underwriter, the third insurer under study, bundles the learning

and the decision in one architecture from the first year: it tracks, corrects, values retention, and prices by the decision, with no rate level.

3 A Stylized Market

The market is the laboratory in which the learning and the decision are measured. It carries 4,000 accounts of middle-market general liability, written by four insurers without reinsurance; Figure 1 maps one year of it, and Appendix A states every assumption with its value, its grounding, and the arms that vary it. Each account has a size and two traits, one of claim frequency and one of claim size, each with a permanent part and a part that drifts from year to year; claims occur, are reported with a lag, and are paid at settlement, with an open claim carried at a case reserve that errs. The claim sizes follow the distribution increased-limits ratemaking uses, a mixed exponential, scaled so that the book's mean claim at the occurrence limit is the industry's published general liability severity, with limits per occurrence and in the aggregate and loss adjustment expense in proportion to the indemnity paid (Appendix A).

Three sources of information exist, and they are not equally shared. A bureau publishes, for each of five classes, the market's pooled loss costs and claim frequency of past policy years. An insurer asked to quote receives the broker's loss run, the account's claims of the past five policy years whoever wrote them, and its underwriter forms a reading of the account of its own, which sets the schedule modification (Appendix A). The insurer that holds an account reads its own claim file for every year it has held it, current to the quarter and with a smaller reserve error than the broker's run carries. The holder therefore knows more about its accounts than a challenger does, and the longer it has held an account the more; nothing else in the market favors an incumbent except the buyer's costs of shopping and switching.

Three conventional insurers compete in every market the paper runs, and they are the benchmark. Each prices by the bureau's plan, the class loss cost times the experience and schedule modifications times its own rate level, with a provision for expenses and profit taken from the industry's published expense ratios, and each sets its level in its own way: the Bureau insurer refiles its multiplier when the indication on its own experience departs from the plan by a stated threshold; the Loss-ratio insurer moves its level every year by the loss-ratio method, with the level of competitors' winning prices as the complement of credibility; and the GLM insurer does the same and adds relativities by class and size band from a generalized linear model fitted to its own book. None files a load on new business, which follows the practice Wu and Lin measure in general liability; each applies a schedule credit on new business in the larger size bands from the winning prices it has seen, inside the filed band of 25 percent either way. The three do not respond to a competitor that learns beyond these rules. Appendices A and B state the plans and the rules in full.

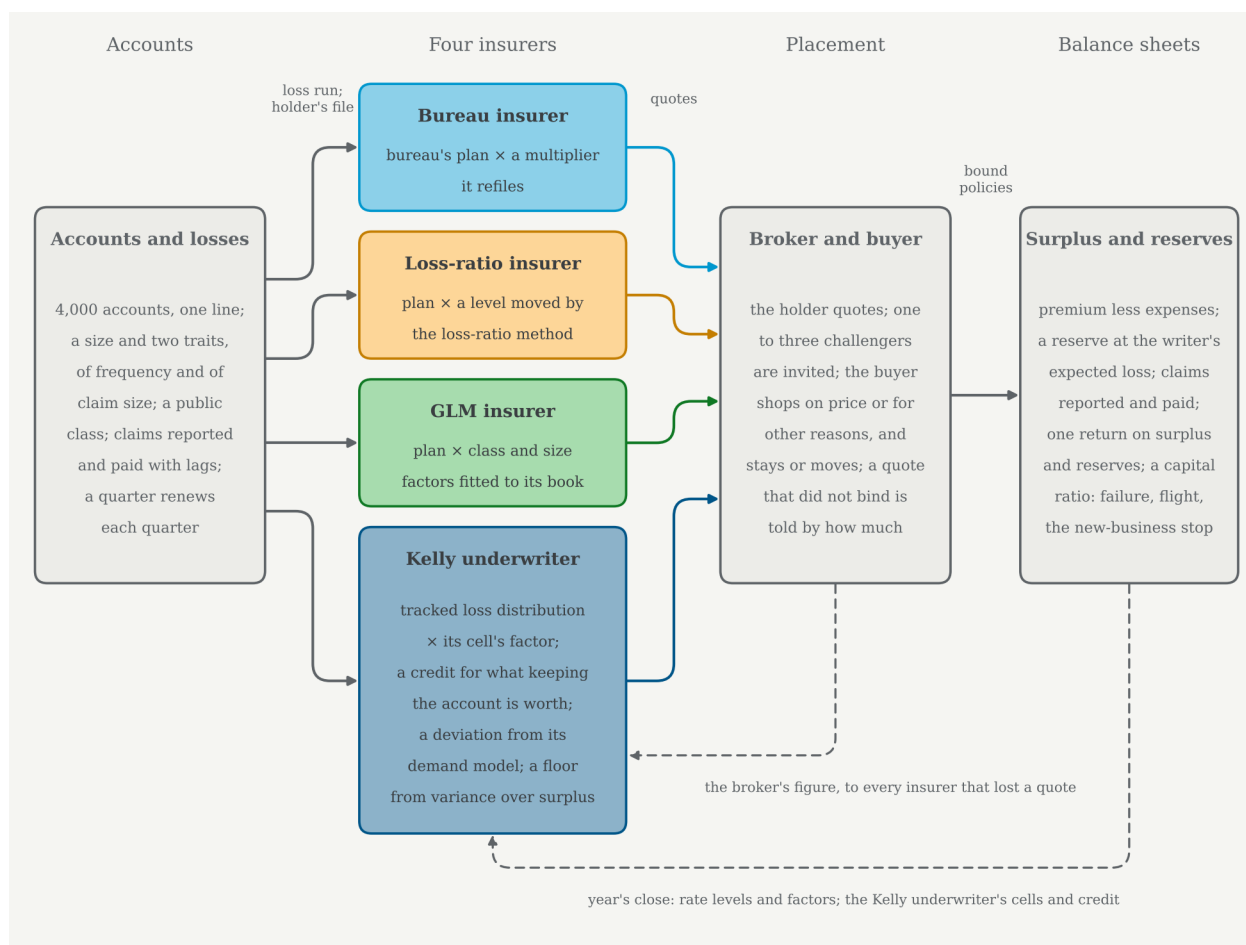


Figure 1: One year of the market: each quarter the holder and challengers quote, the buyer chooses, a losing insurer is told its miss, and the policy is booked; at the year's close levels, cells, and credit are re-estimated and capital is tested (§A.5). Solid arrows are flows within a quarter, dashed arrows what returns to an insurer. In all figures the Bureau insurer is bright blue, the Loss-ratio insurer amber, the GLM insurer green, the fourth seat deep blue. Source: authors' construction.

The buyer and the broker close the market. An account renews once a year; its holder quotes, and the broker invites one or more challengers, more for a larger account. The buyer goes to market when the holder's renewal quote stands far enough above the lowest competing quote to overcome its cost of shopping, and switches when the gap overcomes its cost of switching; the two costs were calibrated, on draws outside the record, so that 14 percent of the accounts of a market of four conventional insurers change insurer a year, half the share of middle-market companies that a survey of the segment found changing an insurer over two years, which gives a premium retention between two published retentions of commercial business (Appendix A). After placement the broker tells an insurer that lost the difference between its quote and the bound premium, rounded; the insurer that bound is told nothing. Every insurer starts the record years with the same endowment, set so that its opening book stands at the industry's aggregate risk-based-capital ratio, and surplus and reserves earn the industry's published investment yield. Capital affects behavior at three thresholds (Appendix C): a broker withholds new-business invitations from an insurer whose capital ratio falls into the band at which regulators watch it, an insurer whose written premium exceeds three

times its surplus stops quoting new business, and an insurer below the level at which regulators take control fails and is replaced. No insurer fails in any replication of any arm of the record.

The books are seeded and the market runs 10 practice years before the 28 record years begin, so that every insurer opens the record with a seasoned book, a file on each account, and estimators that have run. An insurer's *growth* is the logarithm of its surplus at the close of the record years over its endowment, net of the return on funds, so that a surplus that earned that return and nothing else has a growth of zero; the text states it beside as a rate of growth a year and as the multiple of the endowment. The market is run 200 times, each run from one draw of its random streams, a *seed*, and every arm of the record runs on the same 200 seeds. A difference between two fillings of the fourth seat is therefore taken within each seed and averaged, with its t statistic, the mean over its standard error across seeds, stating how precisely the simulation resolves the difference; a difference whose t statistic exceeds 2 in magnitude is one the record resolves. The paper states a difference in growth as a percentage of terminal surplus. Every difference is a total effect: what changes in one seat moves the other insurers' books, rate levels, and capital, and the difference carries their response.

4 What Learning Is Worth

The fourth seat is filled in 14 ways on the same 200 seeds, the three conventional insurers the same in each (Table 1). Four fillings form a two by two: the plan's estimate or the learned estimate of each account's loss distribution, priced as the Loss-ratio insurer prices or by the Kelly decision. With the plan's estimate and the plan's pricing the fourth seat is a second Loss-ratio insurer, and the market has no insurer that learns. With the learned estimate and the plan's pricing it is the plan route. With the plan's estimate and the Kelly decision it applies the decision to the plan's expected loss and to a variance taken from the class. With both it is the Kelly underwriter, the architecture of Section 7, which prices by the decision on its own. The fifth filling is the Kelly route, the learned estimate priced by the Kelly decision as a deviation from the plan route's price, in the construction of record (Table 1, the second row); the other fillings vary the construction of the Kelly route and of the Kelly underwriter: the form of the credit; its discount, at the insurer's own growth or at the return on funds, the *funds discount*, which the theory of the growth-optimal portfolio gives;⁷ and a *pessimism variant* that lowers every chance of keeping an account by one standard deviation of the fit's uncertainty (Appendix F names each).

⁷John B. Long Jr., "The Numeraire Portfolio," *Journal of Financial Economics* 26, no. 1 (1990): 29–69.

Table 1: The fillings of the fourth seat, means over 200 seeds. Growth is the logarithm of the fourth seat's surplus at the close of the 28 record years over its endowment, net of the return on funds; the rate a year is the mean geometric rate of growth of surplus, in percent, and the multiple the terminal surplus over the endowment, both at the mean logarithm. The share is the fourth seat's share of the accounts at the close, and the turnover the share of the market's accounts changing insurer a year. The Kelly underwriter's rows: as registered, with the credit in proportion to the premium quoted; corrected, with the anchored credit. The record's names for the fillings are in Appendix F. Source: authors' calculations.

The fourth seat	Growth, logarithm	Rate a year	Multiple of endowment	Share of accounts	Changing insurer a year
<i>The two routes</i>					
Learning at the plan's pricing	+0.935	3.39	2.55	0.672	0.163
Learning at the Kelly decision, with pessimism, funds discount	+0.945	3.43	2.57	0.494	0.263
Learning at the Kelly decision, funds discount	+0.945	3.43	2.57	0.488	0.263
Learning at the Kelly decision, with pessimism, growth discount	+0.941	3.42	2.56	0.461	0.270
Learning at the Kelly decision, growth discount	+0.939	3.41	2.56	0.452	0.269
<i>The other fillings</i>					
Second Loss-ratio insurer: plan's estimate, plan's pricing	+0.534	1.93	1.71	0.223	0.141
Plan's estimate at the Kelly decision, quote-anchored credit	+0.532	1.92	1.70	0.142	0.291
Learning at the Kelly decision, quote-anchored credit	+0.836	3.03	2.31	0.308	0.258
Kelly underwriter as registered: growth discount, quote-anchored credit	+0.691	2.50	2.00	0.250	0.394
Kelly underwriter, funds discount, quote-anchored credit	+0.678	2.45	1.97	0.259	0.403
Corrected Kelly underwriter, funds discount	+0.711	2.57	2.04	0.338	0.368
Corrected Kelly underwriter, with pessimism, funds discount	+0.723	2.62	2.06	0.353	0.365
Corrected Kelly underwriter, growth discount	+0.745	2.70	2.11	0.332	0.368
Corrected Kelly underwriter, with pessimism, growth discount	+0.748	2.71	2.11	0.333	0.366

Learning is the largest effect on the record. A second Loss-ratio insurer in the fourth seat grows at 1.93 percent a year, 1.71 times its endowment over the record years. The plan route, the learned estimate priced as that insurer prices, grows at 3.39 percent a year, 2.55 times its endowment: learning alone adds 49.2 percent to terminal surplus ($t = +31.10$). In the same market the plan route ends with 71.8, 84.4, and 68.0 percent more terminal surplus than the Bureau insurer, the Loss-ratio insurer, and the GLM insurer beside it, and it holds 67 percent of the market's accounts at the close. The source of the lead is in the business it writes. The new business a conventional insurer wins has a true expected loss of 1.012 times the insurer's own estimate, because an insurer wins where its estimate is low; the new business the plan route wins has a true expected loss of 0.927 times its estimate, below the estimate that won it. With the tracked estimate the insurer does not win mainly where its estimate is wrong. Figure 2 gives every insurer's growth by seed in the four fillings of the two by two.

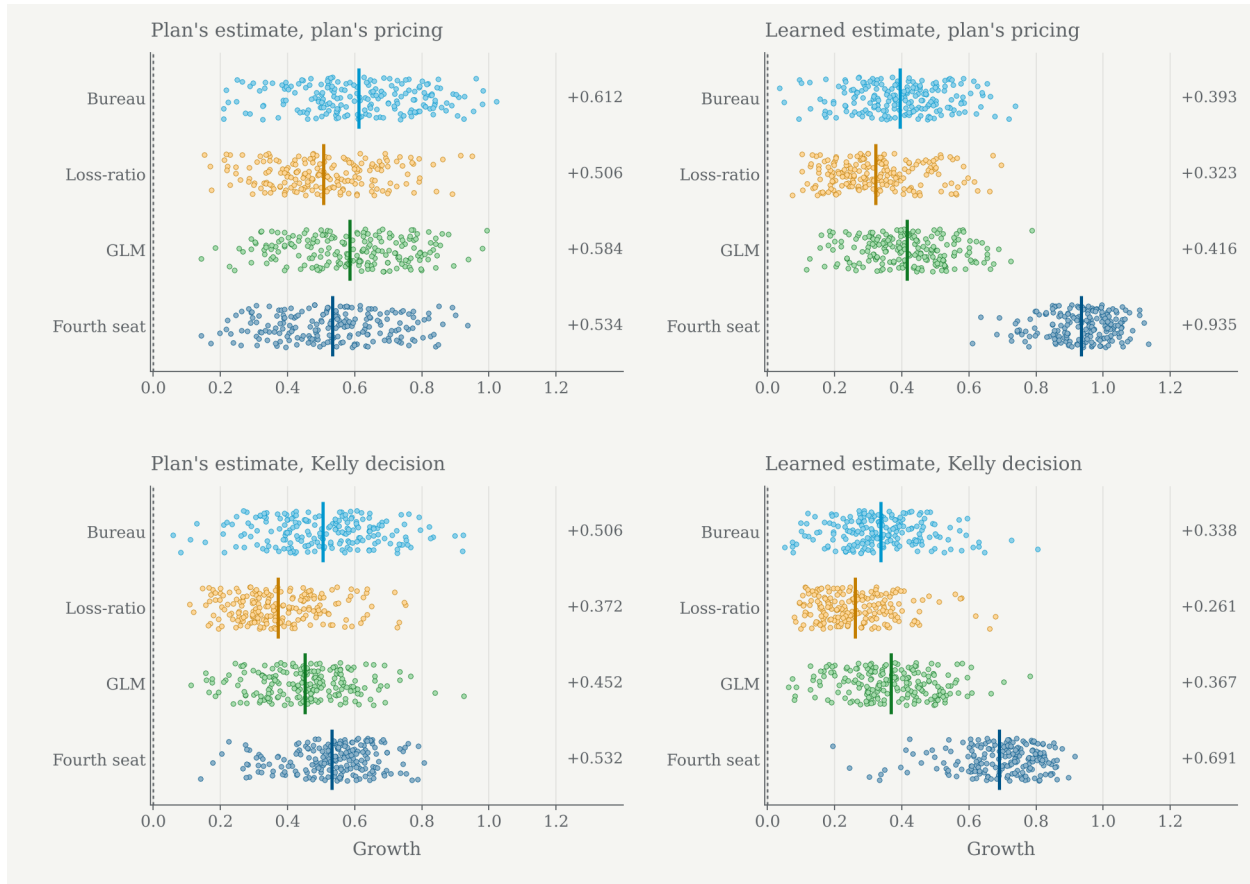


Figure 2: Growth by insurer and seed, over 200 seeds, in the four arms of the two by two, the pricing by row (plan's above, Kelly decision below) and the estimate by column (plan's left, learned right). Each insurer's row of points gives growth, the logarithm of its surplus at the close of the record years over its endowment net of the return on funds, on one horizontal scale, a vertical bar marking the mean, stated at right; zero is short-dashed. Source: authors' calculations.

The Kelly decision adds nothing to an estimate it cannot improve. Applied to the plan's own estimate, with the plan's variance taken from the class, the decision moves terminal surplus by -0.3 percent ($t = -0.24$): a pricing rule is worth what the information behind it is worth, and the plan's estimate carries no more information when a different rule prices it. The Kelly underwriter, which learns and prices by the decision in one architecture, grows at 2.50 percent a year, 2.00 times its endowment, 16.9 percent more terminal surplus than a second Loss-ratio insurer and less than the plan route (Section 7). In the two by two the lead comes from the learning.

The form in which the value of keeping an account enters the decision is worth more than the decision itself. With the anchored credit, the construction of record, the Kelly route reaches the plan route (Section 5); with a credit in proportion to the premium quoted, the form first registered, the same decision on the same estimate falls 9.4 percent of terminal surplus short of the plan route ($t = -12.01$), and the Kelly underwriter as registered, which carries that form, 21.6 percent ($t = -31.11$), against the registered prediction of a gain; the anchoring is worth 11.6 percent of terminal surplus on the Kelly route ($t = +14.85$) and 5.6 percent on the Kelly underwriter ($t = +11.03$).

The anchoring was designed after the first registration's outcomes were on the record, and registered before any run of its arms on the record's seeds; Appendix F states every prediction of the paper, held or failed, and the order in which each was written and run.

5 Two Routes to the Same Growth

The plan route and the Kelly route price the same learned estimate, and they reach the same growth. This section states the equivalence (§5.1) and then how the two routes differ, in the prices they quote, the book each builds, the turnover around them, and the path by which each arrives (§5.2); Table 2 sets the two side by side, and Figure 3 draws the difference between them at the close and year by year.

5.1 Alike in Growth

The Kelly route grows at 3.43 percent a year, 2.57 times its endowment, and the plan route at 3.39 percent, 2.55 times, the two highest growths of any filling of the fourth seat (Table 1). Their difference at the close of the record years, the Kelly route less the plan route paired by seed, is 1.1 percent of terminal surplus, with 113 of 200 seeds above zero and a t statistic of 1.9656, below the threshold at which the record resolves a difference (Figure 3, left). A difference the record does not resolve is not a finding of equality, so the paper tests equivalence against a margin stated before the test: 0.028 in the logarithm over the 28 record years, 0.10 percentage points of the rate of growth a year, chosen in the objective's own unit and below a tenth of what learning alone is worth. Equivalence is shown where the 90 percent interval of the difference lies inside the margin either side of zero. The two routes' growth is shown equivalent at that margin, on each discount of the credit and with or without the variant on the hold curve; the primary pair's interval, the two routes as Table 1 lists them first, is $[+0.0018, +0.0198]$, inside the margin and above zero (Appendix F, Table F.3): at the equivalence test's 90 percent the Kelly route's lead stands, and at the rule that resolves a difference, which asks more, it does not. It does not take the Kelly decision to realize the gain from learning: there is a pricing that computes no Kelly decision at quote time and whose growth lies within the margin of the decision's.

Why the two arrive together is in the construction. Both price the same estimate of each account, and the Kelly route can move a price only inside the filed rate band, which the market sets and the plan route also observes; what the decision chooses within the band changes the book and the timing of the margins more than it changes the surplus at the horizon.

5.2 Where the Two Routes Differ

The routes are told apart in the quote data, in the market around them, and in time (Table 2). The Kelly route prices renewals 12.6 percent above the plan route's price on average, with more than a quarter of its renewal quotes within 0.25 percent of price of the band's upper end, and it prices new business 8.4 percent below that price. The pattern, a low first price recovered on the accounts that stay, is the lowballing the theory of pricing under asymmetric learning predicts of an insurer that knows its own accounts better than its competitors do;⁸ the Kelly route arrives at it without a model of that equilibrium, from the value of keeping an account and the chance of winning it. The

⁸D'Arcy and Doherty, "Adverse Selection"; Jia and Wu, "Insurer Commitment," 89.

Kelly route holds 49 percent of the market's accounts at the close of the record years against the plan route's 67 percent, and the accounts around it change insurer at 26 percent a year against 16: a smaller book at a higher renewal margin, in a market that turns over more.

Table 2: The plan route and the Kelly route side by side, means over 200 seeds; the Kelly route is learning at the Kelly decision with the pessimism variant on the funds discount, the primary arm. Growth a year is the mean geometric rate of growth of surplus; the prices are the Kelly route's mean deviation from the plan route's price, a factor less one; the shares and the turnover are at the close of the record years and per year; the path's rows read the difference between the routes at the close of each record year, paired by seed, with the volatility the standard deviation over the record years of the yearly increment of growth and the dispersion the standard deviation across seeds of growth at the close. Source: authors' calculations.

	The plan route	The Kelly route
<i>The destination</i>		
Growth of surplus, percent a year	3.39	3.43
Terminal surplus over the endowment	2.55	2.57
<i>The prices</i>		
Renewals, percent above the plan route's price	—	12.6
New business, percent below the plan route's price	—	8.4
<i>The book and the market</i>		
Share of the market's accounts held at the close, percent	67	49
Accounts changing insurer a year, percent of accounts	16	26
<i>The path</i>		
Record years through which the Kelly route trails, at t below -2	—	23
Deepest shortfall of the Kelly route, percent of the surplus of that year, in year 10	—	-5.0
Within-path volatility of growth	0.0204	0.0179
Dispersion of growth across the market's draws at the close	0.0956	0.1218

In time the two routes are told apart through most of the record. Read at the close of each earlier record year, the Kelly route trails the plan route by a difference the record resolves in every year through year 23, deepest in year 10 at -5.0 percent of the surplus of that year, and the difference lies inside the margin from year 22 through the close (Figure 3, right); the two tests both hold where they overlap, a difference resolved against zero lying inside a margin wider than it. Within a seed, the median over seeds of the longest run of consecutive years in which the Kelly route trails is 23.5 years. Along any one path the Kelly route's growth is the steadier of the two, its within-path volatility the lower ($t = -14.42$), and across the market's draws its outcomes are the more dispersed, the standard deviation of its growth at the close 1.274 times the plan route's. The two routes reach one destination by paths a reader of quotes, of retention, and of the yearly accounts can tell apart.

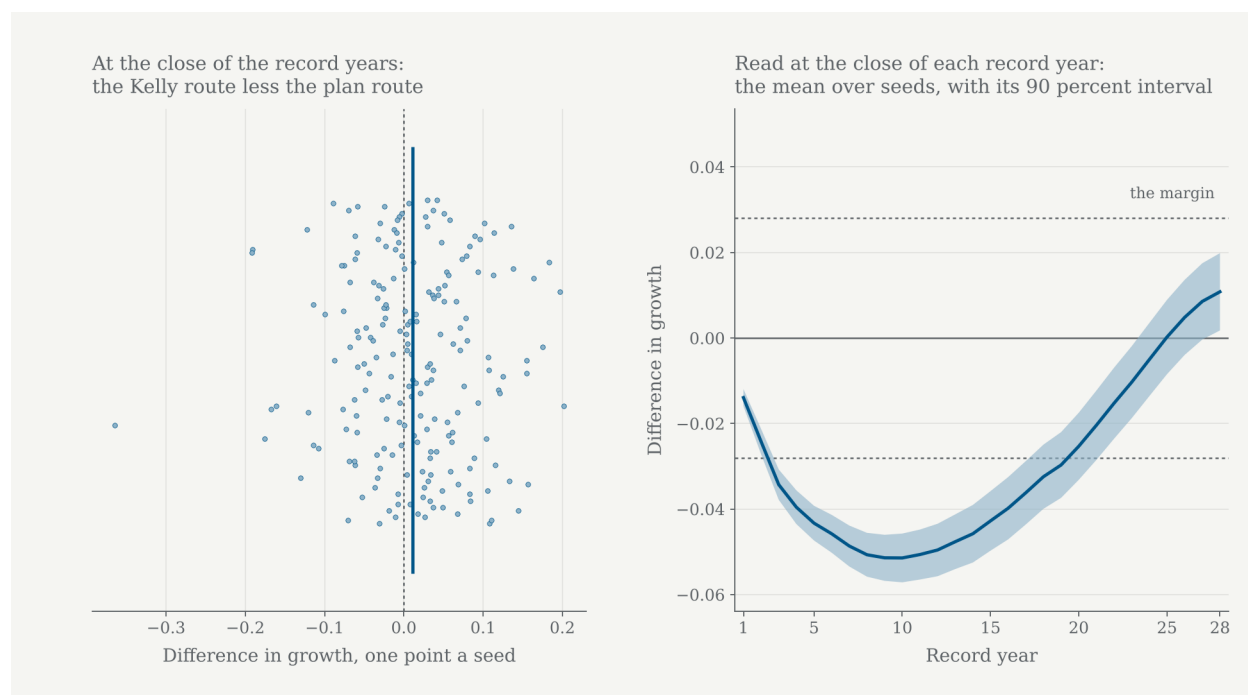


Figure 3: At left, the Kelly route less the plan route, the fourth seat's growth at the close of the record years paired by seed (horizontal axis), one jittered point a seed; the solid bar is the mean and the short-dashed line zero. At right, the same difference read at the close of each record year (horizontal axis), the mean over seeds as the line and its 90 percent interval as the band; the dashed lines are the registered margin of equivalence either side of zero. The Kelly route is learning at the Kelly decision with the pessimism variant on the funds discount. Source: authors' calculations.

6 When the Routes Part

The equality of the two routes is a property of the market the record runs. Two dimensions of the market act on the Kelly decision: the filed rate band, which bounds how far the decision can move a price, and the buyer's responsiveness to price, which sets how much shopping and switching a given price gap produces and so the markup the decision chooses. The two routes were run at one other value of the band and two of the responsiveness, under predictions registered before the runs, and in four further markets run beside the record: a market with a second insurer that learns, in the GLM insurer's seat; a smaller insurer, its endowment set so that its opening book stands at the capital ratio at which broker flight begins; a market of one tenth of the accounts; and a market with a severity trend that no insurer projects, the yearly growth of the severity of other liability occurrence claims from 2015 through 2024,⁹ applied to the mean claim limited per occurrence. Table 3 reports the Kelly route less the plan route in each, in percent of terminal surplus.

⁹Jim Lynch and William Nibbelin, *Increasing Inflation on Liability Insurance – Impact as of Year-End 2024* (Insurance Information Institute and Casualty Actuarial Society, October 2025), 18; for other liability occurrence, net ultimate loss and defense and cost containment expense per reported claim, accident years 2015 through 2024.

Table 3: The Kelly route less the plan route at the close of the record years, the fourth seat's growth paired by seed over 200 seeds, as a percentage of terminal surplus with its t statistic, in the record's market and in seven others that move one dimension of it. The Kelly route is learning at the Kelly decision with the pessimism variant on the funds discount; the plan route is learning at the plan's pricing in the same market. Appendix F names the registration each row belongs to, and Appendix I gives each market's other contrasts. Source: authors' calculations.

The Kelly route less the plan route	Percent of terminal surplus	t
The record's market	+1.1	+1.97
A rate band of 40 percent either way, in place of 25	-14.2	-30.28
A buyer more responsive to price (scale 0.025 in place of 0.05)	-3.4	-5.56
A buyer less responsive to price (scale 0.075)	+3.1	+5.51
A second insurer that learns, in the GLM insurer's seat	+0.3	+0.44
A smaller insurer: the endowment at 0.467 of the record's	+2.3	+3.19
A market of one tenth of the accounts	+12.3	+8.50
A severity trend of 6.8 percent a year that no insurer projects	+7.2	+4.89

Where the band is wide the Kelly route trails. At a band of 40 percent either way the Kelly route ends 14.2 percent of terminal surplus below the plan route ($t = -30.28$), beyond the margin; both routes grow less at the wider band than at the record's, and the record does not attribute the Kelly route's loss there between its own quotes and the conventional insurers' wider schedule credits. The buyer's responsiveness moves the routes apart in either direction: a buyer more responsive to price moves the Kelly route 3.4 percent behind the plan route ($t = -5.56$), and a buyer less responsive moves it 3.1 percent ahead ($t = +5.51$), each a difference the record resolves, whose interval crosses the margin. The decision's quotes move as the buyer's responsiveness changes, the renewal deviation and the share of renewal quotes at the band's upper end lower where the buyer is more responsive and higher where it is less (Appendix I).

Where the market is smaller, the accounts fewer, or the severity trend rising, the Kelly route leads. Beside a second insurer that learns, the two routes stay within the margin of each other (+0.3 percent). In the smaller insurer the Kelly route leads the plan route by 2.3 percent of terminal surplus ($t = +3.19$), in the market of one tenth of the accounts by 12.3 percent ($t = +8.50$), and under the severity trend by 7.2 percent ($t = +4.89$), beyond the margin. In each of the four further markets the plan route outgrows a second Loss-ratio insurer, as in the record's market: the gain from learning stands wherever the market is run, and what moves between markets is the worth of the decision on top of it.

The two by two keeps its pattern at five settings of two dimensions that do not act on the decision, the Bureau insurer's refiling threshold and the noise of an underwriter's reading of an account: learning alone adds to growth at every setting, and the Kelly decision on the plan's estimate adds at two of the five, by at most 4.5 percent of terminal surplus (Appendix F, Table F.4).

7 The Kelly Underwriter

The Kelly underwriter learns and prices by the decision in one architecture from the first year: it tracks each account from the files it reads, corrects by cell for the business it wins, values retention from its own book, and quotes by the decision around a working price set at the plan's provision,

with no rate level (Appendix E states the computation step by step). It outgrows the three conventional insurers and trails both routes. Figure 4 draws the market around it over the record years, the conventional insurers' shares of the accounts and their rate levels, with the Kelly underwriter in the fourth seat and with a second Loss-ratio insurer in its place.

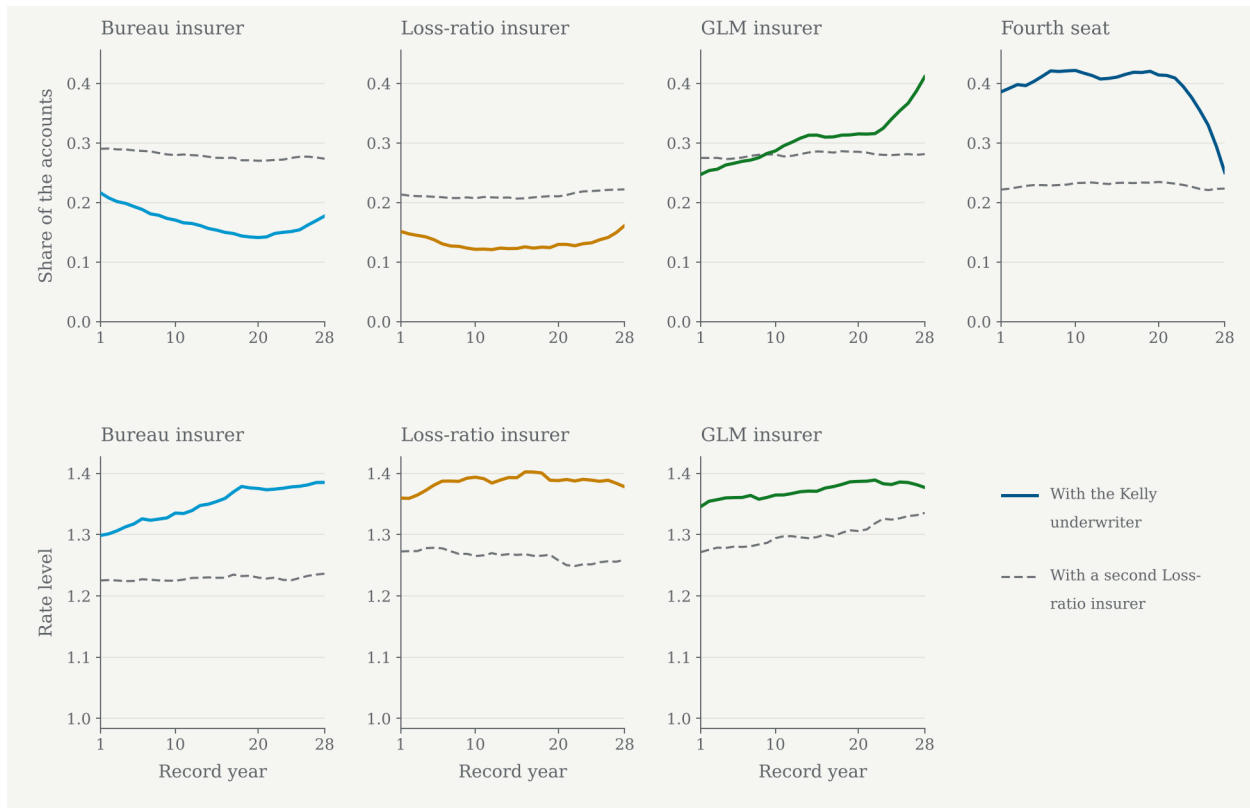


Figure 4: Account shares (upper row) and conventional insurers' rate levels (lower row) at the close of each record year (horizontal axis), means over 200 seeds; a level multiplies the bureau's loss costs, one at the practice years' start. Solid lines in the insurer's color are the market with the Kelly underwriter, dashed gray lines that with a second Loss-ratio insurer as fourth seat. Each row has one vertical scale; the lower row's fourth cell holds the legend. Source: authors' calculations.

In the base arm, the market with the Kelly underwriter in the fourth seat as the first registration ran it, with the credit in proportion to the premium quoted, it grows at 2.50 percent a year, 2.00 times its endowment, and ends with 42.4, 53.7, and 38.2 percent more terminal surplus than the Bureau insurer, the Loss-ratio insurer, and the GLM insurer beside it (t of 22.73, 30.25, and 22.35). With the anchored credit it trails the plan route by 17.3 percent of terminal surplus on the growth discount, the discount at which it was registered ($t = -30.88$), and by 20.0 percent on the funds discount. The two run the same estimator of each account's distribution on different books and files; on the Kelly underwriter's side the two differ in the working premium, the cells, the credit's levels, and the deviation chosen on the hold curve, and on the plan route's side in the rate level and the schedule credit on new business. The record does not attribute the gap among these parts.

Removing the Kelly underwriter's parts one at a time shows which it cannot do without (Appendix G). Without the renewal credit its growth falls by 20.1 percent of terminal surplus ($t = -29.91$) and its leads over all three conventional insurers turn negative: the value of keeping

an account is what makes the Kelly underwriter bid for accounts it does not hold, and it is the one part whose absence turns the leads. Without the hold curve or without the cells' correction the growth is lower and the leads stand. Removing the three at once raises growth by 8.0 percent of terminal surplus ($t = +7.84$), the fourth seat then pricing the tracked estimate at the plan's provision with no decision and holding 93 percent of the accounts at the close; the parts do not add, because the hold curve's deviation and the cells' factors raise prices that the credit lowers.

Around the Kelly underwriter the market changes (Figure 4). It prices new business below its renewals in every size band and by more the larger the account, by 26.2, 31.5, and 36.8 percent of its renewal price in the smallest, the middle, and the largest band, through the credit, the cells' factors on new business, and the deviations chosen on the hold curve. Accounts change insurer at 39 percent a year against 14 in the market with a second Loss-ratio insurer in its place; the Kelly underwriter holds 25 percent of the accounts at the close; and the three conventional insurers end the record years at higher rate levels and lower growth than in the market without it, each still earning more than the return on its surplus. Its share of the accounts, 42 percent at its height, falls in each of the final nine record years to 25 percent at the close, the years in which the value of keeping an account falls toward zero at the horizon; the record does not attribute the fall (Appendix G). Its leads over the conventional insurers hold across a program of 79 arms that change one condition, rule, threshold, or constant of the market at a time, 236 of 237 leads above zero at a t above 2, the one exception a lead over a Bureau insurer that holds a fixed rate level, indistinguishable from zero; under the severity trend it trails all three (Appendix H).

8 What Follows for an Insurer, and Conclusion

This section states what each course leads to (§8.1), the limits of the findings (§8.2), and the conclusion (§8.3).

8.1 What Each Course Leads To

An insurer that learns each account's loss distribution from its own files and prices through its rating plan reaches the highest growth of surplus on the record, shared with the Kelly route within the margin of equivalence. Its pricing architecture does not change: the plan's class loss cost, experience and schedule modifications, and expense and profit provisions stand, and the learned estimate enters where the plan's estimate stood. Its book is the largest of any filling of the fourth seat, 67 percent of the market's accounts at the close of the record years, and the Kelly route trails it through the first two decades.

An insurer that learns the same and prices by the Kelly decision reaches the same growth by a different book and a different path. It quotes renewals above the plan's price and new business below it, it holds half the market's accounts at the close, its renewal margin is the higher, and the market around it turns over more. For most of the record years its surplus trails the plan route's; the two meet only over the final years. Where the market is smaller, the accounts fewer, or the severity trend rising, this route leads; where the rate band is wide or the buyer responsive to price, it trails. The form in which the value of keeping an account enters the price decides whether the decision adds or costs, and the record's answer is the anchored credit.

An insurer that builds the learning and the decision into one architecture, estimating each account from its own files alone and pricing by the decision from the first year, outgrows the three

conventional insurers in the record's market and across the sensitivity program with one exception, trails them under the severity trend, and trails both routes in the record's market and in each of the four further markets. The value of keeping an account is the part it cannot do without. The difference between it and the plan route is not attributed among its parts on this record.

8.2 Limits

The market of the record is one line of business with independent traits, no loss trend (Section 6 reports a market with one), no common shock, no reinsurance, no underwriting cycle,¹⁰ and at most two entrants. The three conventional insurers are built to price as the sources describe a rating plan, a loss-ratio indication, and a generalized linear model; what they do not have is a response to a competitor that learns beyond the schedule credit and the complement of competitors' rates, and an insurer in their place might acquire one. The market with a second seat that learns gives the Kelly underwriter a competitor that learns from the start, not one that responds to it. The buyer responds to price alone, through a demand model calibrated to one target. The schedule credits on new business and the deviations of the Kelly route and the Kelly underwriter answer competitors' prices, which the rule cited for the filed band excludes from what a schedule may rate; for personal lines, a task force of the National Association of Insurance Commissioners considers rate adjustments by a buyer's price elasticity of demand or propensity to shop inconsistent with the standard that rates not be unfairly discriminatory.¹¹ The model follows the practice the sources report and does not assess compliance.

The two routes are run at the record's calibration of the rate band and the buyer's responsiveness and at one other value of the band and two of the responsiveness, and the equivalence of their growth is a statement within a registered margin, at this record's precision, and at the close of the record years. The decision's variance term, the criterion's weight on risk, is small at this market's surplus; of the record's market and the four run beside it, it is largest in the market of one tenth of the accounts, at 0.6421 percent of the expected loss of an account at the curve's own scale of claim sizes, and removing it is shown not to move the growth of three of the Kelly constructions by more than a stated amount (Appendix D). The record therefore tests the criterion's rule for pricing under uncertainty about the odds, a margin against the chance of holding the account and the discounted value of keeping it, and not the property that separates the criterion from the maximization of expected profit when the stake is large against the surplus. The record runs the full criterion and no fraction of it, the form that trades growth for security, and the criterion's long-run growth does not make it the choice of an investor whose preferences are not logarithmic.¹² The horizon is known to the insurer, the value of keeping an account falls toward zero over the final record years, and growth is read at that horizon; the record does not measure a market whose horizon ends earlier, or one under a going-concern horizon. And every difference is a total effect within a market that responds through its competitors' capital and books, measured over 200 seeds of one model, so that its t statistic is a statement about the simulation's precision at that count and not about an observed

¹⁰On the explanations of underwriting cycles: Mary A. Weiss, "Underwriting Cycles: A Synthesis and Further Directions," *Journal of Insurance Issues* 30, no. 1 (2007): 31–45.

¹¹National Association of Insurance Commissioners, Casualty Actuarial and Statistical (C) Task Force, *Price Optimization White Paper* (adopted by the task force, November 19, 2015), 15–16.

¹²Paul A. Samuelson, "Why We Should Not Make Mean Log of Wealth Big Though Years to Act Are Long," *Journal of Banking and Finance* 3, no. 4 (1979): 305–307; L. C. MacLean, W. T. Ziemba, and G. Blazenko, "Growth versus Security in Dynamic Investment Analysis," *Management Science* 38, no. 11 (1992): 1562–1585; Leonard C. MacLean, Edward O. Thorp, and William T. Ziemba, eds., *The Kelly Capital Growth Investment Criterion: Theory and Practice* (Singapore: World Scientific, 2011).

market; the registered rule is applied to each prediction without adjustment for their number, and Appendix F reports the count and the reading that adjusts for it.

8.3 Conclusion

A rating plan tells an insurer what to charge. The Kelly criterion tells it what the charge is for: the growth of its surplus, which the criterion maximizes in the long run, and which requires, beyond the expected loss a plan also estimates, three things a plan does not hold, the variability of the loss against surplus, the value of keeping the account, and the chance that the quote wins. An insurer can learn all four from the records of its own operations, and this paper has measured what the learning is worth and how it is best put to work, in one market, over 200 replications, under predictions registered before the runs that tested them.

The learning is the largest single effect in the record's market. An insurer that estimates each account's loss distribution from its own files and prices it through the rating plan it already has grows its surplus at 3.39 percent a year against 1.93 for an insurer that does not learn, and it reaches the highest growth of surplus on the record, shared with the Kelly route. An insurer that prices the same estimate by the Kelly decision reaches the same growth, within a margin set before the test. The criterion supplies the objective, and the objective is met by either route; the decision is not required to meet it. What the decision changes is the book and the path: a smaller book with a higher renewal margin, new business priced below the plan, more turnover in the market, and two decades of trailing before the two routes converge. In the markets where the Kelly route pulls ahead, a smaller insurer, fewer accounts, a rising severity trend, it does so by the same means. The one architecture that bundles learning and decision from the first year outgrows every conventional insurer but under a rising severity trend, and trails both routes.

The learning of this paper is econometric: credibility weighting at estimated moments, a correction by cell, and a hold curve fitted to the quotes won and lost, each reading the data an insurer holds. The same data admit a deeper learner. A neural network identifies features, nonlinearities, and dependencies among an account's characteristics, its claims, and the market's responses that estimators of a stated form do not, and the capacity of such networks to represent any smooth relation is established.¹³ The two routes of this paper carry over to such a learner unchanged: whatever estimates each account's loss distribution and its chance of being won can be priced through the plan or by the decision, and the record's measure of what the learning is worth is the measure that learner would be held to. What the record shows is that the growth the Kelly criterion names can be reached by an insurer that never computes the Kelly decision, once it has learned what the decision would need, and that the decision, formed with the value of keeping an account anchored at the plan's price, reaches it too, along a path a reader of quotes and retention can tell from the other. What the criterion's weight on risk would add in a market whose accounts are large against its surplus, and how the two routes fare where all three conventional insurers learn, are questions this record leaves open.

¹³Kurt Hornik, Maxwell Stinchcombe, and Halbert White, "Multilayer Feedforward Networks Are Universal Approximators," *Neural Networks* 2, no. 5 (1989): 359–366.

A The Market: Its Assumptions, Its Insurers, and Its Loss Process

This appendix states the market of Section 3 in full: its assumptions (Table A.1), the accounts and their losses, what an insurer knows, the three traditional insurers (the body's conventional insurers) and how each prices, the buyer and the broker, capital and the measure of growth, and the loss process.

A.1 Accounts and Losses

The market holds 4,000 accounts of one line, written by four insurers without reinsurance. An account has a size, lognormal with median 1 and a log standard deviation of 1, and two traits, one of claim frequency and one of claim size, independent of each other. Each trait, in logarithms, is the sum of a permanent part and a part that drifts as a first-order autoregression with persistence $\rho = 0.85$: the frequency trait's permanent part has a standard deviation of 0.40 and its yearly shock one of 0.30, and the severity trait's permanent part one of 0.632 (a variance of 0.40) and the same shock. In a policy year the account has a Poisson number of claims with mean λ , the product of its size, its frequency trait, and one expected claim a year at the median size. A claim's size is mixed exponential, the form used in increased-limits ratemaking, with means of \$9,277, \$92,766, \$927,662, and \$9,276,615 at weights of 0.700, 0.250, 0.045, and 0.005, scaled by the account's severity trait; the means are set, through one factor of 1.85532 on a stated base curve, so that the book's mean claim limited at the occurrence limit is Verisk's general liability indemnity severity of \$56,213 a claim.¹⁴ Indemnity is limited to \$1,000,000 an occurrence and \$2,000,000 in the aggregate of a policy year, and loss adjustment expense is 0.39156 of the indemnity paid, in addition to the limit, the ratio of Verisk's loss adjustment expense severity of \$22,011 to its indemnity severity. A claim is reported with a lag, 0.55 of a policy year's claims by the end of the year and all of them within four years, and is paid once, at settlement, up to 12 years after it is reported. Until it settles it stands on an insurer's books at a case reserve, its ultimate value times a lognormal error. §A.6 states the process in full.

A.2 What an Insurer Knows

Three sources of information exist, and they are not equally shared. The first is public. A classification score, the sum of an account's two permanent traits read with noise, places each account in one of five classes, and a bureau publishes for each class the market's pooled experience of five policy years ending two years before the rating year: a loss cost, a loss cost with each occurrence and its expense capped at \$100,000, and a claim frequency, each per unit of size. The second is the submission. An insurer that is asked to quote receives the broker's loss run, the account's claims of the past five policy years whoever wrote them, valued one quarter before the policy's inception,¹⁵ with an open claim at a case reserve whose error has a log standard deviation of 0.30. Each insurer's underwriter also forms a reading of the account of its own, the permanent traits read with

¹⁴The form: Clive L. Keatinge, "Modeling Losses with the Mixed Exponential Distribution," *Proceedings of the Casualty Actuarial Society* 86 (1999): 654–698; Joseph M. Palmer, *Increased Limits Ratemaking for Liability Insurance* (Casualty Actuarial Society study note, 2006). The severity: Verisk, *2024 Executive Insights: General Liability*, 1st ed., Verisk's Core Lines Services Report, Second-Quarter 2019 to First-Quarter 2024 (Verisk, 2024), 4.

¹⁵Association for Cooperative Operations Research and Development (ACORD), *Commercial Insurance Application*, ACORD 125 (2007/10), 3, which asks for the claims or losses of the prior five years.

noise of the size of the score's, a standard deviation 0.7 times that of the traits. The third is private. The insurer that holds an account reads its own claim file for every year it has held it, current to the quarter and with half the case-reserve error, and keeps the loss run it received when it acquired the account. The holder therefore knows more about its accounts than a challenger does, and the longer it has held an account the more: this is the asymmetry of the literature on dynamic insurance contracts, and nothing else in the market favors an incumbent except the buyer's costs of shopping and switching.

A.3 Three Traditional Insurers

Each traditional insurer prices by the bureau's plan and differs from the others in how it sets its level. For an account of class c and size z the plan's expected loss is

$$\bar{x} = L_c z \times M_{\text{exp}} \times M_{\text{sch}} \times R, \quad (\text{A.1})$$

where L_c is the bureau's loss cost, M_{exp} the experience modification, M_{sch} the schedule modification, and R the insurer's rate level, and the premium is $\bar{x}/(1 - \pi)$ with a provision $\pi = 0.34$ for expenses of $\varepsilon = 0.30$ of premium and a profit of 0.04.¹⁶ The experience modification weights the account's capped losses against the class's by credibility: $M_{\text{exp}} = 1 + Z_{\text{exp}}(A - E_{\text{rep}})/E$, with A the account's losses on the file the insurer reads, each occurrence with its expense capped, E the class's expected capped losses for the period, E_{rep} their expected reported part, and $Z_{\text{exp}} = E/(E + K)$. The bureau's plan as filed uses three policy years, the latest year before the rating year left out, and K of \$66,000: the plan states K as \$100,000 of premium, and it enters on the expected-loss basis, times $1 - \pi$, so that a change of the provision moves K with it. An insurer applies the plan's formula over every year its file holds, the latest again left out, a year of age j weighted by d^j , and estimates d and K from its own book by least squares (Appendix B). The schedule modification is what the underwriter's own reading adds to the class, within a filed band of 25 percent either way.¹⁷

The *Bureau insurer* files a multiplier on the bureau's loss costs and refiles it when the indication on its own experience reaches the threshold below. At each year's close it computes the loss-ratio method's indication on its own experience of the past five policy years: with a its reported losses, e the expected reported losses it priced those years at, brought to its present level, and n its reported claims, the indicated change is

$$Zf + (1 - Z)\psi, \quad f = a/e, \quad Z = \min(1, \sqrt{n/1,082}), \quad (\text{A.2})$$

with the complement of credibility ψ equal to one.¹⁸ The standard of 1,082 claims is the full-credibility standard of limited-fluctuation credibility for claim frequency, applied here to losses, as practice often applies it. Where the indicated change differs from one by 0.10 or more it refiles, and its level is multiplied by the change; otherwise its level stands.

¹⁶National Association of Insurance Commissioners (NAIC), *Report on Profitability by Line by State in 2024* (Kansas City, Missouri: NAIC, 2026), 9: for other liability countrywide, a line wider than general liability, general expense, selling expense, and taxes, licenses, and fees of 8.2, 18.8, and 1.9 percent of net premiums earned, 28.9 percent in all.

¹⁷Fisher et al., *Individual Risk Rating*; a limit of 25 percent either way is that of Florida Administrative Code, Rule 69O-170.004. The same rule, at (2)(b), excludes from the characteristics a schedule may rate "the degree of competition for the risk or the rates which may be offered by other insurers"; the schedule credit on new business described below and the Kelly underwriter's deviation within the band (§D.1.4) answer competitors' prices, as underwriters use schedule rating to meet a competitor's quote in practice, and the model follows the practice: Lisa A. Hays, "Managing Commercial Lines Pricing Levels in a Loss Cost Environment," *Casualty Actuarial Society Forum* (Winter 2002): 11.

¹⁸Laurence H. Longley-Cook, "An Introduction to Credibility Theory," *Proceedings of the Casualty Actuarial Society* 49 (1962): 194–221; Werner and Modlin, *Basic Ratemaking*, chaps. 8 and 12.

The *Loss-ratio insurer* moves its level every year by Equation A.2 on a window of five policy years, with the complement ψ set to competitors' rates: the level of the winning prices relative to its own plan prices, fitted as a censored normal to the broker's figures on the quotes it lost (§A.4) and to the quotes it bound (Appendix B). The complement takes that level once the window holds 100 lost quotes, and one before.

The *GLM insurer* adds relativities of its own to the plan. Each year it fits a multiplicative model with a logarithmic link to the capped reported losses of its own book over five policy years, with a factor for each class and for each of three size bands, thirds of the year's accounts by manual premium, the premium at the bureau's loss cost for the account's class and size before the experience and schedule modifications, and takes each fitted factor at its classical credibility; its level moves by the ratio of its book's reported losses to their expected amount at its present relativities, taken at the classical credibility of the book's claims (Appendix B).¹⁹

On new business, a quote on an account it does not hold, a traditional insurer files no load. The measured fact is that new business runs at a higher loss ratio than renewals in every one of Wu and Lin's books. In their two books of general liability, new policies receive schedule debits about half as often as renewals and schedule credits as often: there, new business is priced in practice no higher than the plan states, and the loss-ratio difference is a cost the price does not carry. Wu and Lin also judge a surcharge on new business justifiable on their data, so a filed load of zero follows the practice their general liability books show and not their reading of what the data justify.²⁰ Filed loads of 0.05 and 0.1 are arms of the record (Appendix H). What an underwriter may move without a filing is the schedule modification, and insurers do not always implement the rate their experience indicates.²¹ In the middle and the largest band the insurer therefore applies a schedule credit or debit to its new-business quotes, equal to the level of the winning prices it has seen in the band relative to their level in the smallest band, from the broker's figures of the past three years, with the credit and the schedule modification together kept inside the filed band. The smallest band is the reference and carries no credit. The rule refers to account size only through the choice of the smallest band as the reference; the load that results in the larger bands is an outcome (§G.2), and three other rules for the load are arms of the record (Appendix H).

A.4 The Buyer and the Broker

An account renews once a year, in the quarter of its inception. Its holder quotes. The broker invites one challenger, drawn at random among the other insurers that are open to new business, and each further one with a probability equal to the account's rank by manual premium among the year's accounts, from zero for the smallest through one for the largest, so that the smallest account draws one competing quote and the largest draws all three. Let Δ be the logarithm of the holder's quote over the lowest invited quote. With probability 0.10 the account is remarketed for reasons unrelated to price and chooses among all the quotes by a logit on log price with the scale s of the response below. Otherwise it goes to market with probability $1/(1 + \exp(-(\Delta - k)/s))$ and, once there, stays with its holder with probability $1/(1 + \exp((\Delta - k')/s))$, taking the lowest invited quote if it does not; k is the cost of shopping, k' the cost of switching, and $s = 0.05$ the scale of the response, each in units of log price. The two costs are equal and calibrated rather than stated. On seeds outside the record, $k = k'$ was moved until 0.14 of the accounts of the market of four plan-pricing

¹⁹Robert A. Bailey, "Insurance Rates with Minimum Bias," *Proceedings of the Casualty Actuarial Society* 50 (1963): 4–11; Werner and Modlin, *Basic Ratemaking*, chap. 10.

²⁰Wu and Lin, "Persistence and Renewal Discounts," 398, 402, and 405.

²¹Werner and Modlin, *Basic Ratemaking*, chap. 13; Hays, "Commercial Lines Pricing Levels," 11.

insurers changed insurer a year, half the 28 percent of middle-market companies that Greenwich Associates found changing an insurer over two years; the halving counts no company changing twice, and independent yearly switching would give 0.151. The calibration gives $k = k' = 0.061184$. At that cost, in the calibration's own run, 0.1404 of the accounts change insurer a year (standard error over 20 seeds 0.0033), 0.2654 go to market, and 0.8313 of premium is retained, between two published retentions of commercial business.²²

After placement the broker tells an insurer that quoted on an account that went to market and did not bind the difference between its quote and the bound premium, as a fraction of the bound premium rounded to the nearest 0.05. The insurer that bound is told nothing, and nothing is said of an account that did not go to market. This is the only information about competitors' prices that any insurer receives.

A.5 Capital, Solvency, and the Measure

Within a seed every insurer starts the first record year with the same endowment, set so that a seat's expected written premium of the opening book and the steady-state reserves of a book of that premium stand at the industry's aggregate risk-based-capital (RBC) ratio of 6.42458 (Appendix C); over the seeds of the record it averages \$556.7 million, from \$519.3 through \$605.8 million.²³ A policy's premium is booked with expenses of 0.30 and a reserve at the writer's own expected loss; the reserve runs off as claims are reported and paid; and surplus and reserves alike earn the *return on funds*, $r = 0.0356$ a year, the industry's published investment yield.²⁴ At each year's close an RBC ratio is computed for each insurer, its surplus over a requirement of the form of the NAIC in its written premium and its reserves (Appendix C). Capital affects behavior at three thresholds. The broker reads the ratio: below 3 it withholds a new-business invitation with a probability that rises to one at 2 (broker flight), and a renewal is never affected. The insurer reads its own writings: in a quarter that opens with its written premium of the year above three times its surplus, the level at which the NAIC's Insurance Regulatory Information System (IRIS) ratio of premium to surplus is flagged, it quotes no new business, and it keeps quoting renewals. And below a ratio of 0.70, or with its surplus exhausted, the insurer fails and is replaced by an entrant, a Bureau insurer with the endowment, at most two times in a seed.²⁵ Table A.1 collects the assumptions of this section.

²²Greenwich Associates, *Insurance Market Pulse: Looking at the Small Business and Middle Market Segments* (2007), the share of middle-market companies that changed a property-casualty or employee-benefits insurer over the two years before the survey, as reported in Douglas McLeod, "Changing Insurers Yields Surprise," *Business Insurance*, June 17, 2007. The retentions: The Hartford Insurance Group, Inc., *Annual Report on Form 10-K for the Fiscal Year Ended December 31, 2025* (filed with the U.S. Securities and Exchange Commission, February 20, 2026), a middle-market premium retention of 83 percent; The Travelers Companies, Inc., "Travelers Reports Excellent Second Quarter and Year-to-Date Results," press release, July 17, 2026, a retention of 86 percent in its Business Insurance segment, retention being the premium available for renewal that was retained, excluding rate and exposure changes, with the segment's National Accounts left out.

²³National Association of Insurance Commissioners, "Aggregated P&C RBC Data: 2025 Data as of June 1, 2026," *Summary: Aggregate P/C RBC Results by Year* (NAIC, 2026).

²⁴National Association of Insurance Commissioners, *U.S. Property & Casualty and Title Insurance Industries—2025 Full Year Results* (NAIC Financial Regulatory Services, 2026).

²⁵The failure threshold is the level at which the NAIC's model act mandates control: National Association of Insurance Commissioners, *Risk-Based Capital (RBC) for Insurers Model Act*, Model 312 (2012).

Table A.1: The market’s assumptions, each with its value in the base arm, its grounding, and the arms that vary it. An assumption is grounded in a source, a calibration to a sourced target, a source’s form, the objective, or the simulation’s scale, or it is stated, where no source was found and the value is the model’s own. An arm moves one assumption, except that the profit and expense arms move K with the provision, since K enters on the expected-loss basis (§A.3). Amounts are per occurrence or per policy year as stated and rates per year; the sources are in the text. Source: authors’ construction.

Assumption	Value	Grounding	Varied
Account size, log standard deviation	1	stated	two arms
Claims a year at the median size	1	stated	two arms
Frequency trait: spread, persistence, shock	0.40, 0.85, 0.30	stated	six arms
Severity trait: spread (variance), persistence, shock	0.632 (0.40), 0.85, 0.30	stated	six arms
Claim size, at Verisk’s severity	mixed exponential	Keatinge; Palmer; Verisk	scale, three arms
Loss adjustment expense over indemnity	0.39156	Verisk	two arms
Limits: occurrence; aggregate (\$)	1,000,000; 2,000,000	Palmer	no
Share of claims reported by year	0.55, 0.85, 0.95, 1.00	stated	two arms
Case-reserve error: loss run; holder’s file	0.30; 0.15	stated	four arms
Classes; size bands	5; 3	scale	no
Noise of a read over the traits’ spread	0.7	stated	two arms
Bureau’s statement: years; lag	5; 2	stated	three arms
Loss run: years; valuation lag, quarters	5; 1	ACORD; stated	years, two arms
Full credibility, claims	1,082	Longley-Cook, frequency	no
Filed experience plan: years; cap; K (\$)	3; 100,000; 66,000	bureau plan’s form	one arm
Schedule band, either way	0.25	Florida rule	one arm
Provision π ; expenses ε ; profit	0.34; 0.30; 0.04	NAIC; stated	three arms
Filed load on new business	0.00	Wu and Lin, GL books	two arms
The load’s rule	schedule credit by band	stated	three arms
Lost quotes to fit: complement; schedule	100; 30	stated	no
Windows: loss-ratio method; GLM fit, years	5; 5	Werner and Modlin; stated	two arms; no
Cells’ window; credit’s; schedule fit’s, years	3; 3; 3	stated	six arms
Bureau insurer: experience, years; threshold	5; 0.10	stated	four arms
Costs of shopping and switching, $k = k'$	0.061184	calibrated to Greenwich	two arms
Scale of the buyer’s response s	0.05	stated	two arms
Remarketing unrelated to price	0.10	stated	two arms
Invitations: power of the account’s rank	1	stated	two arms
Broker’s figure rounded to	0.05	stated	two arms
Capital requirement: ρ ; κ	1.72456; 0.0968742	NAIC 2025	no
Flight band, RBC ratio (NAIC trend test)	2 through 3	NAIC; slope stated	two arms
New-business stop, premium over surplus	3	NAIC IRIS; the stop stated	one arm
Failure, RBC ratio	0.70	NAIC Model Act	no
Return on funds, on surplus and reserves	0.0356	NAIC yield	two arms
Opening capital, RBC ratio	6.42458	NAIC 2025	no
Entrants after failures, at most	2	stated	no
Demand model: prior spreads; decay	0.25, 1.0; 0.98	stated	four arms
Renewal credit: discount, gross; horizon	own growth; final year	the construction’s choice (Section 4); the objective	the discount, \$5
Accounts; years of seeding, practice, record	4,000; 19, 10, 28	scale	no
Seeding: holders changing a year	0.15	stated	no

The books are seeded over 19 years in which accounts change holder at random at a rate of 0.15 a year, so that every insurer opens with a seasoned book and a file on each account. The market then runs 10 practice years, in which every surplus is reset to the endowment at the year’s close while rate levels, estimators, and books carry forward, and 28 record years; at the close of the final

practice year, before the reset, each insurer's reserves on the business it wrote before the record years are restated, the provision for unreported claims at its true expectation and each open claim's case reserve at what the claim will pay (Appendix C). An insurer's *growth* is

$$g = \ln \frac{S_T}{S_0} - T \ln(1 + r), \quad T = 28, \quad r = 0.0356, \quad (\text{A.3})$$

the terminal logarithm of its surplus over the endowment, net of the return on funds, so that g is zero for a surplus that earns that return and nothing else, and g/T is a yearly rate of growth from writing business, the investment income on the reserves an insurer holds included. The terminal surplus S_T is taken from the insurer's own books at the close of the final record year, its reserves at its own estimates: no restatement like that of the final practice year is made there, and the paper does not measure the difference one would make. An insurer that fails leaves the market, and its growth is not defined; no insurer fails in any seed of any arm of the record, so every mean and contrast is over all 200 seeds, and the record does not test the decision's exposure to ruin. Every arm of the record runs on seeds 0 through 199, and a contrast between two arms or two insurers is paired by seed, with its mean, its t statistic (the mean over its standard error, the sample standard deviation over seeds divided by the square root of the number of seeds), and the number of seeds in which it is above zero. A contrast between two arms is a total effect: what changes in one seat moves the other insurers' books, rate levels, and capital, and the contrast carries their response; it is not the effect of the change with the competitors held as they were.

A.6 The Loss Process

An account i has a size z_i , lognormal with median 1 and log standard deviation 1. Its frequency trait in year t is $u_{it} = \alpha_i + d_{it}$, with α_i normal with mean zero and standard deviation 0.40 and $d_{it} = \rho d_{i,t-1} + \eta_{it}$, $\rho = 0.85$, η_{it} normal with standard deviation 0.30, started from its stationary distribution. Its severity trait is of the same form, with a permanent standard deviation of 0.632 (a variance of 0.40) and the same persistence and shock, independent of the frequency trait. The stationary variance of a trait's drifting part is $0.30^2/(1 - \rho^2)$, so the frequency trait has a variance of 0.484 and the severity trait one of 0.724. Each trait enters as its exponential less half its variance, so that it has a mean of one across accounts in every year.

In policy year t the account's claim count is Poisson with mean

$$\lambda_{it} = \lambda_0 z_i \exp(u_{it} - \tfrac{1}{2} \text{Var}(u)), \quad \lambda_0 = 1. \quad (\text{A.4})$$

A claim's size before any limit is exponential with a mean drawn from the four means of §A.1 at their weights and multiplied by the account's severity trait; the curve's mean before any limit is \$117,813, and limited to \$1,000,000 an occurrence it is \$61,964. Within a policy year the aggregate limit of \$2,000,000 erodes in the order in which the claims are paid. Loss adjustment expense is 0.39156 of the indemnity paid and is outside the limits. An account's true expected loss in a year, the quantity the tables call by that name, is λ_{it} times the expected claim limited per occurrence at the account's severity trait, with loss adjustment expense, times the share of the year's expected occurrence-limited losses that the aggregate limit leaves.

A claim occurs in a quarter of the policy year drawn uniformly. It is reported within the policy year with probability 0.55, by the end of the second year with probability 0.85, of the third with 0.95, and of the fourth with certainty: in the policy year, in a quarter drawn uniformly from its occurrence quarter through the year's final quarter, and in a later year uniformly over that year's

quarters. It settles between one quarter and 12 years after it is reported, 0.30 of claims within a year of report, 0.50 within two, 0.82 within five, and all within 12. An open claim is carried at a case reserve, its ultimate value times $\exp(\sigma_c \xi - \sigma_c^2/2)$ with ξ standard normal, one draw a claim, and σ_c of 0.30 on a broker's loss run and 0.15 on the holder's own file. A settled claim stands at what was paid.

B The Rating Plans

The classification score is α_i of the frequency trait plus the permanent part of the severity trait plus normal noise with a standard deviation 0.7 times that of the sum, and the classes are the quintiles of its distribution. For policies of year t the bureau's statement covers policy years $t - 6$ through $t - 2$, valued at the start of year t . For each class it gives the reported losses, the reported losses with each occurrence and its expense capped at \$100,000, and the reported claims, each divided by the public expected reported share of its year and summed over the market, per unit of size; a class's figure is weighted with the all-class figure at the classical credibility of the class's reported claims.

In the experience modification, for an account of class c and size z let $B_c z$ be the expected capped loss of a year. Over the years of the file the insurer reads, a year of age j (the latest year of the period at age zero, the period ending two years before the rating year) weighted by $w_j = d^j$,

$$M_{\text{exp}} = 1 + \frac{E}{E + K} \cdot \frac{A - E_{\text{rep}}}{E}, \quad A = \sum_j w_j A_j, \quad E = B_c z \sum_j w_j, \quad E_{\text{rep}} = B_c z \sum_j w_j F_j, \quad (\text{B.1})$$

with A_j the year's capped reported losses on the file and F_j the public expected reported share of the year at the file's valuation. The holder applies it to its own claim file and a challenger to the broker's loss run. Each insurer estimates (d, K) once a year by least squares over a grid, d from 0.5 through 1.0 and K from \$25,000 through \$500,000: for the accounts it held and renewed in the latest expired policy year it compares what each pair would have given as that year's modification, from the file as it stood when the year was quoted, with the year's capped losses as now reported, and it adds the weighted squared differences to those of the years before. Until its first estimate it uses the plan as filed, $d = 1$ over three years and K of \$66,000.

In the schedule modification, with (ν_1, s_1^2) the mean and variance of the permanent traits' sum in the account's class and (ν_2, s_2^2) its mean and variance given the class and the underwriter's reading, each under a normal approximation, the modification is $\exp(\nu_2 - \nu_1 + (s_2^2 - s_1^2)/2)$, held within 1 ± 0.25 .

On the policies the GLM insurer wrote in the five policy years ending at the year's close, it fits $A \approx E \times f_{\text{class}} \times f_{\text{band}} \times f_{\text{new}}$, with A a policy's capped reported losses, E their expected reported amount at its present relativities, and f_{new} a factor by band on new business, by iterative proportional fitting, the estimator of a Poisson likelihood with a logarithmic link. The band factors and then the class factors are brought to a mean of one, weighted by E . The change of level is the ratio of the book's reported losses over those years, uncapped and with expense, to their expected reported amount at the present relativities. The level, each class relativity, and each band relativity are then multiplied by $Zf + 1 - Z$, with f the change of level or the fitted factor and Z the classical credibility of the claims behind it. The factors on new business do not enter its price under the base rule for the load.

For the schedule credit on new business, the insurer takes for each band, from its quotes as a challenger of the past three years on accounts that went to market and were placed, the logarithm

of the bound premium over its own plan price at its present level: observed where the broker gave a figure, and known only to be at or above its quote where the quote bound. A normal distribution is fitted by maximum likelihood to both kinds of observation, and the band's level of winning prices is the fitted mean plus half the fitted variance. The credit in the middle and the largest band is the exponential of the band's level less the smallest band's. A band's credit changes only where the band and the smallest band each hold 30 lost quotes in the window; otherwise it keeps its previous credit. The same fit over all bands, on its own window of five years, gives the Loss-ratio insurer's complement of credibility ψ once it has 100 lost quotes in the window.

C The Capital Standard

At a year's close an insurer with written premium W in the year, reserves Q , and surplus S has the risk-based-capital ratio

$$\text{RBC} = \frac{S}{\kappa \sqrt{W^2 + (\rho Q)^2}}, \quad \rho = 1.72456, \quad \kappa = 0.0968742. \quad (\text{C.1})$$

The square-root combination is the form of the NAIC's formula, in which the premium-risk and reserve-risk charges combine under a root; the model carries those two charges and absorbs the others in κ in the industry's proportion, the covariance term of the two underwriting charges alone standing at 0.470 of the industry's Authorized Control Level. The two constants are calibrated on one year of the NAIC's aggregated data: ρ is the reserve charge per dollar of reserves over the premium charge per dollar of net written premium, and κ is set so that the industry's own point—net premium at 0.773 of surplus, reserves at 1.05664 times premium—stands at the industry's aggregate ratio of 6.42458.²⁶ A failed insurer's seat (§A.5) passes, while entries remain, to an entrant at a level of one with the endowment and no book. The model insurer's reserves run above the industry's all-lines ratio to premium, as a long-tail book's do, so at an equal ratio of premium to surplus its RBC ratio is lower than the industry's, as the formula's reserve charge makes it.

Broker flight is the broker's response to the ratio. At each year's close the broker reads each insurer's RBC ratio and, in the following year, extends a new-business invitation to it with probability $\min(\max(\text{RBC} - 2, 0), 1)$: one at a ratio of 3 or more, falling linearly to zero at 2, account by account by its own random stream. The band is the NAIC trend-test band and stands in the model as a proxy for the financial-strength thresholds brokers act on; the steepness of the withdrawal has no source, and the band shifted upward and flight removed are arms of the record. Flight is suspended in the practice years, and the first record year reads the final practice year's closing statement with the surplus reset to the endowment.

The stop on new business (§A.5) reads IRIS ratio 2, net premiums written over policyholders' surplus, at the level the NAIC flags, on the year's written premium to date; the test is made again each quarter against the surplus of the moment, and the year's closing ratio can end above the threshold on renewals alone.²⁷ The stop removed is an arm of the record.

The endowment (§A.5) is set so that the opening book's expected written premium W and the steady-state reserves of a book of that premium, $Q = (1 - \pi) \tau W$ with τ the mean time from booking to payment (§D.1.2), stand at the industry's aggregate RBC ratio, which gives an opening ratio of

²⁶National Association of Insurance Commissioners, "Aggregated P&C RBC Data"; National Association of Insurance Commissioners, 2025 *Full Year Results*.

²⁷National Association of Insurance Commissioners, "NAIC IRIS Ratios: Property and Casualty, Data Year 2025" (NAIC, May 19, 2026).

written premium to surplus of about a third in every year. Surplus changes within a quarter by the premium written less expenses and less the reserve set up at the writer's expected loss, by the release of that reserve along the public reporting pattern, by the case reserves set up on the claims reported in the quarter and released at their settlement, by the payments made, and by investment income on surplus and reserves (§A.5). In a practice year no insurer fails; the restatement of the reserves at the close of the final practice year (§A.5) takes the provision for unreported claims at its true expectation, not conditioned on the claims already reported.

D The Kelly Decision and the Estimators

This appendix states the approach of Section 2 with its equations: the objective and the decision (§D.1), what is learned, from what data, and by what estimator (§D.2), the estimators in detail (§D.3), and the decision's variance term (§D.4).

D.1 The Objective and the Decision

This section states the objective and the decision that follows from it: the criterion (§D.1.1), the increment one account adds to expected growth (§D.1.2), what keeping an account is worth (§D.1.3), the decision, the chance of winning at a price weighed against the increment, with the lowest premium at which the insurer writes (§D.1.4), and the form in which the value of keeping an account enters the decision, with its correction (§D.1.5). Each quantity is stated with the data it reads, and Appendix E gives the computation in the order in which it is carried out.

D.1.1 The Objective

Let S be the insurer's surplus. The Kelly criterion is the objective: the insurer chooses its actions to maximize the expected logarithm of its surplus at the horizon, $\mathbb{E}[\ln S_T]$. Since $\ln S_T - \ln S_0$ is T times the mean geometric rate of growth of surplus over the horizon, this is the maximization of the expected mean geometric rate of growth of surplus. Where an action leaves the insurer's later opportunities unchanged, the objective is the maximization of $\mathbb{E}[\ln S']$, the expected logarithm of next period's surplus, in each period; where it changes them, as when an account written today is held in later years (§D.1.3), the one-period objective carries what the action leaves for later. Two features of an insurer's position change the statement of the criterion's property for a bettor who knows the odds (Section 1). The insurer does not choose a stake at given odds: it quotes a price, the buyer decides, and the account is written whole or not at all. And the insurer does not know the distribution of an account's loss; it has an estimate. The decision below approximates the criterion for the distribution the insurer is given: to second order in the account's share of surplus, one account and one year at a time, with the value of later years entered to first order (§D.1.3). The property of optimal growth is not claimed for this approximation in this market. The rest of this section states each of the four quantities the decision needs and how it enters the price.

D.1.2 The Increment of One Account

Consider one account with loss X , estimated mean $\hat{x} = \mathbb{E}[X \mid \text{data}]$, and estimated variance $\hat{v}$, quoted at premium P . Written, it changes surplus by $n - X$, where $n = P(1 - \varepsilon)$ is the premium net of expenses. Expanding the logarithm to second order in the account's share of surplus, the

account taken alone against the current surplus, and dropping the square of the expected margin, which is small against the variance,

$$\mathbb{E} \left[\ln \frac{S + n - X}{S} \right] \approx \frac{n - \hat{x}}{S} - \frac{\hat{v}}{2S^2}, \quad (\text{D.1})$$

which is a mean–variance rule in which the weight on variance, one half over the surplus, is fixed by the objective and is not a parameter. Two further terms enter the increment. The premium is held against the policy until its claims are paid and earns the return on funds meanwhile: with $\tau = 4.083$ years the mean time from booking to payment under the market’s reporting and settlement patterns, the expected loss net of that return is $\hat{x}(1 - c_x)$ with $c_x = r\tau = 0.1454$, the return over the mean time to payment without compounding. And in a market where an account written today is likely to be held in later years, the increment carries what keeping the account is worth, V per unit of net premium, the renewal credit of §D.1.3. The increment of the decision is

$$G(P) = \frac{n(1 + V) - \hat{x}(1 - c_x)}{S} - \frac{\hat{v}}{2S^2}. \quad (\text{D.2})$$

In the market of Section 3 the variance term is small. At the endowment’s mean over the seeds, \$556.7 million, it is 0.0641 percent of the expected loss for an account at the curve’s own scale of claim sizes, and since the expected loss and the variance both rise in proportion to the claim rate, it does not vary with the account’s size. Without the term, three of the record’s Kelly constructions choose another deviation on 0.018 to 0.029 of the quotes they compute, and removing it is shown not to move their mean growth by more than 0.010 (§D.4).

The Kelly underwriter’s working price starts from the same expected loss at a plan’s provision, the *working premium* $\hat{x}/(1 - \pi)$, so that with nothing else learned it charges what a plan charges on its own estimate. On new business the credit enters the working premium, which solves $P(1 - \pi) + V_{\text{nb}} P(1 - \varepsilon) = \hat{x}$, with V_{nb} the credit on new business (Equation D.3): the insurer asks of a new account this year’s provision less what the account is expected to return in later years, and the divisor applies to a negative credit as it does to a positive one. An account worth acquiring is thus quoted below the price the same estimate has as a renewal: the divisor nets the credit from the plan’s provision, a rule of this construction, and the decision of §D.1.4 chooses the deviation around the price so set. In the literature on dynamic insurance contracts “lowballing” names a first premium below the actuarially fair one, recovered on the accounts that stay.²⁸ The Kelly underwriter writes new business at 1.28 times its true expected loss (§G.2), which net of its expenses of 0.30 of premium is below that loss and above it discounted at the return on funds over the mean time to payment.

D.1.3 What Keeping an Account Is Worth

Equation D.1 values an account by one year’s margin. In a market where the buyer bears a cost of switching and the holder knows the account better than a challenger does, an account written today is likely to be held next year, and the objective, which is growth over the whole horizon, counts the margins of the years to come. The objective also fixes how they are discounted. A margin m that an account adds k years ahead raises $\ln S_T$ by about m/S_{t+k} , and where the insurer’s surplus grows at its own yearly rate $\hat{g}$, gross of the return on funds, $S_{t+k} \approx S_t e^{\hat{g}k}$, the margin counts about $e^{-\hat{g}k}$ in units of today’s surplus: the discount factor is $\beta = e^{-\hat{g}}$. The step replaces the expectation of

²⁸D’Arcy and Doherty, “Adverse Selection”; Jia and Wu, “Insurer Commitment,” 89.

S_t/S_{t+k} by its value on the path of mean growth, which by Jensen's inequality understates it; the funds discount of Section 4 takes another first-order route, which discounts at the return on funds. A margin that falls after the final record year adds nothing to $\ln S_T$, so the sum runs through the horizon H , the number of years from the year after the one quoted through the final record year. With p_1 and m_1 the retention and the expected margin per unit of net premium at an account's first renewal, and p_s and m_s at later renewals,

$$V_{\text{ren}}(H) = m_s \sum_{k=1}^H (\beta p_s)^k \quad \text{and} \quad V_{\text{nb}}(H) = \beta p_1 [m_1 + V_{\text{ren}}(H-1)], \quad (\text{D.3})$$

with $V_{\text{nb}}(0) = 0$. This is the *renewal credit*: for an account the insurer holds, the discounted margins of the renewals to come; for an account it does not, the first renewal's margin and what follows it, reached with probability p_1 . The five quantities are estimated on the insurer's own book over the credit's window (three years): the retentions in accounts, the margins as Equation D.2's numerator without the credit per unit of net premium, on first renewals kept and on later renewals kept apart—the margin of a newly won account's first renewal is the margin of a selected account, which is what the decision needs—and $\hat{g}$ as the window's mean yearly log growth of surplus, the return on funds included, each year's growth taken from the surplus at its opening to that at its close and the window reading the practice years as it reads the record years. The derivation carries no cap and no stated discount rate: the discount is the insurer's own growth, a negative margin gives a negative credit, and a finite horizon, here the number of record years, needs no bound for any $\hat{g}$ (a going-concern sum exists only where $\beta p_s < 1$). Until the window holds a first renewal, the credit on new business is zero.

D.1.4 The Chance of Winning, and the Decision

A quote is a bid, and the increment $G(P)$ is earned only if the quote holds the account. Let $h(\ell)$ be the probability that a quote at log deviation ℓ from the working premium holds. The expected increment of quoting is $h(\ell) G(P_\ell)$ with P_ℓ the working premium times e^ℓ , and the Kelly decision is the deviation that maximizes it:

$$\ell^* = \arg \max_{\ell} h(\ell) G(P_\ell), \quad P = \max \left\{ P_{\ell^*}, \frac{\hat{x}(1 - c_x) + \hat{v}/(2S)}{(1 - \varepsilon)(1 + V)} \right\}. \quad (\text{D.4})$$

A higher deviation earns more on an account that is held and holds fewer accounts; the two are weighed by the insurer's own hold curve, taken as the posterior average that §D.2.3 states. Because the Kelly underwriter rates on a schedule plan as the others do, its deviation and its schedule modification together stay inside the filed band of 25 percent either way, so that ℓ^* ranges over the band and is at most $\ln 1.25 = 0.223$ on an account whose schedule modification is one. The second element of Equation D.4 is the *floor*: the premium at which Equation D.2 is zero, the lowest at which writing the account adds to expected growth, with the credit of the account's phase, renewal or new business, inside it. The floor rises with the account's variance, falls as surplus grows, and falls with the value of keeping the account, so that of two accounts with one expected loss the one with the wider distribution is quoted no lower, and an insurer charges more for risk when its surplus is low. Without the credit and the return on the premium held, the floor's premium net of expenses is the expected loss plus a loading on the variance of one over twice the surplus, the premium of the variance principle at that loading; the fair premium of a discounted-cash-flow approach to rate

regulation and the allocation of capital among lines of business are two other constructions of an insurer's charge for risk.²⁹

Equation D.4, over the filed band around the working premium, is the decision. Its inputs other than the surplus S , the expense share ε , and the return on the premium held, c_x , are $\hat{x}$ and $\hat{v}$ for each account, the credits V_{nb} and V_{ren} , the hold curve h , the provision π , and the band. A rating plan supplies a version of $\hat{x}$, the provision, and the band; it supplies none of the others. An insurer may hold a learned estimate and price it as a plan prices, or price it by the decision; the paper names the first *learning at the plan's pricing* and the second *learning at the Kelly decision*, and Section 5 measures the two, the decision there applied as a deviation from the plan's pricing of the learned estimate.

D.1.5 The Form of the Credit, and Its Correction

Equation D.2 carries the credit as V per unit of the net premium n . In the decision of §D.1.4 the premium varies with the deviation under consideration, and the form in which the credit follows it is a choice of construction with a measured consequence. In the form first registered, the *quote-anchored credit*, the credit enters the increment as $Vn(\ell)$, with $n(\ell)$ the net premium of the quote at deviation ℓ , in proportion to the premium of the deviation under consideration, so that the gradient of the objective carries V times the premium's own derivative: a markup earns continuation value in proportion to its size. The continuation of a won account is invariant to today's deviation to first order: next year's quote is next year's rule applied to next year's estimate, and the buyer's decision to shop compares the holder's renewal quote with the lowest invited quote (§A.4), which excludes the holder's own raise. The invariance is argued from the market's rules; the study does not measure each won account's continuation by the deviation that won it. What today's deviation buys is today's margin and the chance of keeping the account at all: the hold probability $h(\ell)$ of Equation D.4 multiplies the whole increment, so the value of keeping the account, $h(\ell) V n(0)$, is carried in full. The construction of record therefore anchors the credit at the *undeviated premium*, the premium of the zero deviation: for the Kelly underwriter, the working premium, and for learning at the Kelly decision (Section 4), the price of the plan's pricing it deviates from. This is the *anchored credit*, $Vn(0)$ in place of $Vn(\ell)$. It removes the gradient of the quote-anchored form, and at a deviation other than zero it also changes the credit's level. The quote-anchored form is the first registration's, and the correction was made after that registration's predictions had failed; Section 5 measures what each form is worth, and Appendix F states the order in which the forms were registered and run.

The anchored form has a stated limit. The policy's own renewals are written above the zero deviation on average, so a credit anchored there understates the credit's level by the policy's mean renewal deviation factor, and since the hold probability multiplies the whole increment, an understated level biases the chosen deviations upward relative to a credit at the realized scale. An anchor at the realized scale would make the credit a function of the deviation it prices, the self-reference the anchor removes, so the understatement is a limit of the construction: the deviations of Section 5 are, if anything, higher than a credit at the realized scale would give, and the record does not measure by how much.

²⁹Hans Bühlmann, *Mathematical Methods in Risk Theory* (Berlin: Springer-Verlag, 1970), chap. 4; Stewart C. Myers and Richard A. Cohn, "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," in *Fair Rate of Return in Property-Liability Insurance*, ed. J. David Cummins and Scott E. Harrington (Boston: Kluwer-Nijhoff, 1987), 55–78; J. David Cummins, "Allocation of Capital in the Insurance Industry," *Risk Management and Insurance Review* 3, no. 1 (2000): 7–27.

D.2 Learning

This section states what the Kelly underwriter learns and how: each account's loss distribution, tracked from the files it reads (§D.2.1); a correction for the business it wins (§D.2.2); and the hold curve, from the broker's figures (§D.2.3). The renewal credit's five quantities are estimated on its own book and are stated with the others in Appendix E. Section D.2.4 reports what was learned in the base arm. Every estimator reads only data that an insurer in the market of Section 3 holds, and what the market treats as public: the bureau's curve of claim sizes and its moments at scale one, the reporting and settlement patterns (§A.1, §A.6), the spread of the case-reserve error on a broker's loss run and on a holder's own file (§A.2, §A.6), the share of loss adjustment expense (§A.1), and, as for every insurer, the distribution of the traits within a class that the schedule modification reads (Appendix B).

D.2.1 The Account's Loss Distribution

A rating plan reads an account's history through one number, the experience modification on its capped losses. The Kelly underwriter tracks two quantities for each account, its claim frequency relative to its class and the scale of its claim sizes, and it does so by the best linear predictor of credibility theory, at moments it estimates, in a form that allows for a risk that changes over time.³⁰ For each year of a file it forms two observations: the reported claim count over its expected number at the class's frequency, and the mean logarithm of the reported claim sizes less the curve's mean log size, an open claim's case reserve corrected for its error. Its estimate of the coming year's frequency relativity and log scale is the best linear predictor from these observations jointly, in which the weight on a year depends on how much the year's observation carries (the expected claims behind it) and on how far back it lies.

The predictor needs the second moments of the two traits: how much of an account's departure from its class stays and how much decays, and at what rate. The Kelly underwriter estimates them. From every file and loss run it has read it keeps the autocovariance of each observation at lags of one through 20 years, fits to each a constant plus a geometrically decaying term, a part that stays and a part that decays with a persistence, and keeps one covariance between the two traits (§D.3). On an account it holds it reads its own file, every year it has held the account and the loss run from before; on an account it is asked to quote it reads the broker's loss run. The expected loss is then the class's claim frequency times the account's size, times the tracked relativity, times the expected limited claim at the tracked scale, with loss adjustment expense, and the variance is that of a compound Poisson sum at the same rate and scale, which leaves out the variance from the uncertainty of the rate and the part of the scale's uncertainty common to the account's claims:

$$\hat{x}_0 = \hat{\lambda} \mathbb{E}[\min(Y, u)] (1 + a), \quad \hat{v} = \hat{\lambda} \mathbb{E}[\min(Y, u)^2] (1 + a)^2, \quad (\text{D.5})$$

with $\hat{\lambda}$ the tracked claim rate, Y the claim size at the tracked scale, u the occurrence limit, and a the loss adjustment expense's share. The underwriter's schedule modification multiplies $\hat{x}_0$ as it does a plan's estimate, and $\hat{v}$ with it, as the cell's factor of §D.2.2 does. The schedule modification and the tracked estimate both read the account's permanent traits, the first through the underwriter's reading and the second through the account's claims; their product is not adjusted for what the

³⁰The predictor, for a risk that does not change over time: Hans Bühlmann, "Experience Rating and Credibility," *ASTIN Bulletin* 4, no. 3 (1967): 199–207. A risk that changes over time: Howard C. Mahler, "An Example of Credibility and Shifting Risk Parameters," *Proceedings of the Casualty Actuarial Society* 77 (1990): 225–308.

two share, and the cells correct its level on the business written. The variance is the quantity a plan does not hold; it enters the price through the increment of Equation D.2, which Equation D.4 multiplies by the hold probability and maximizes, and through the floor of Equation D.4.

D.2.2 The Business Won

An insurer bears losses only on the accounts it wins, and winning is informative. Where several insurers quote from estimates with error and the buyer favors the lowest price, an insurer wins more often where its own estimate is low, so the losses of the business won run above the estimates that won it.³¹ Whether the effect moves a market's prices has been tested: in general liability insurance, Harrington and Danzon test whether insurers' differing forecasts of claim costs explain the low prices of the years before the liability crisis of the mid-1980s, and they find the evidence consistent with low pricing from moral hazard and not from differing information.³² The size of the effect depends on how many insurers quote and on what each knows, and it differs between an account the insurer holds and one it does not. The Kelly underwriter measures it. It sorts the policies it has written into four *cells*, one of renewals and three of new business, one in each size band, and at each year's close applies the loss-ratio method to each cell: with a_c the reported losses of the cell's policies of the past three policy years, e_c the expected reported losses on the tracked estimates $\hat{x}_0$, and Z_c the classical credibility of the cell's reported claims, the cell's factor becomes

$$\kappa_c \leftarrow Z_c \frac{a_c}{e_c} + (1 - Z_c) \kappa_c, \quad (\text{D.6})$$

and the expected loss in the price is $\hat{x} = \kappa_c \hat{x}_0$ for an account quoted in cell c , the variance taking the same factor. The cells take the place of a rate level: the Kelly underwriter has no multiplier on its whole book, and its estimate is brought to its experience cell by cell.

D.2.3 The Demand Model

The hold curve h of Equation D.4 is learned from the only data on competitors' prices an insurer has, the broker's figure. The figure is censored: it is given on the quotes that did not bind and withheld on those that did. Let ζ be a latent log price: the logarithm of the premium at which the account would bind with another insurer, over the Kelly underwriter's working premium for it, each observation brought on-level to the reference of its *segment*, a size band in renewals or in new business, before a fit (§D.3), as the traditional insurers bring their quotes on-level. On a quote that was lost, ζ is observed, the quote's own deviation less the figure. Every quote that bound enters as the event $\zeta \geq \ell$, since its holder is told nothing, including a renewal kept against a lower competing quote and one kept without going to market, which the market may also have undercut; the buyer's costs of shopping and switching are thus absorbed in the fitted μ_ζ , and h is a fitted hold curve rather than the distribution of where the market stands. The Kelly underwriter models ζ as normal, $\zeta \sim N(\mu_\zeta, \sigma^2)$, with a stated prior: $\mu_\zeta \sim N(0, 0.25^2)$, the market centered on the insurer's own working premium with the spread of the filed band, and $\ln \sigma \sim N(\ln 0.10, 1.0^2)$. The posterior is taken at the mode of the decayed log likelihood plus the log prior, the lost quotes entering by

³¹Edward C. Capen, Robert V. Clapp, and William M. Campbell, "Competitive Bidding in High-Risk Situations," *Journal of Petroleum Technology* 23, no. 6 (1971): 641–653; Paul R. Milgrom and Robert J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica* 50, no. 5 (1982): 1089–1122.

³²Scott E. Harrington and Patricia M. Danzon, "Price Cutting in Liability Insurance Markets," *Journal of Business* 67, no. 4 (1994): 511–538.

their density and the bound quotes by the probability of the event, with weights that decline by 2 percent a quarter, and its covariance from the inverse of the Hessian of the negative log posterior at the mode, a Laplace approximation.

The decision then uses the hold probability averaged over that posterior,

$$\bar{h}(\ell) = \int \left[1 - \Phi\left(\frac{\ell - \mu_\zeta}{\sigma}\right) \right] \pi(\mu_\zeta, \sigma \mid \text{data}) d(\mu_\zeta, \sigma), \quad (\text{D.7})$$

exactly over μ_ζ given $\ln \sigma$ and by a 12-node Gauss–Hermite rule over $\ln \sigma$. For one period the average is what the objective implies under uncertainty about the curve’s parameters: the increment G of Equation D.2 does not depend on them, so the expected increment of a deviation is the posterior-mean hold probability times G . The decision does not value what a deviation teaches about the curve in later quarters, and its deviations carry no rule that keeps them dispersed. In den Boer and Zwart’s setting such a rule is an interval around the average of past prices, shrinking over time, that the price chosen on the current estimate may not enter, and under it the optimal price is eventually learned.³³ Before any observation the prior predictive governs, so a quote can leave the working premium from the first quarter. The curve is fitted separately in each size band for renewals and for new business, six segments in all.

D.2.4 What Was Learned

Table D.1 reports the learned quantities in the base arm, as means over the record years and the seeds.

The cells’ factors are above one in every cell (Table D.1): the losses of the business the Kelly underwriter writes run above the tracked estimates, and the factor on new business falls with account size. One reading, which no arm isolates, is that a loss run of five years carries more claims on a large account than on a small one. After the cells’ correction the true expected loss of the new business the Kelly underwriter wins still exceeds its estimate, by 5.9 percent in the smallest band and by 0.8 percent in the largest. The record carries the estimate’s accuracy on the business written, as here and through the cells, and not on every account quoted. In a single year a cell with few claims reaches a factor as high as 4.64 (Table D.1).

On its retained renewals the Kelly underwriter earns a margin of 0.272 of premium net of expenses at an account’s first renewal and 0.287 at later ones; it keeps 0.385 of its accounts at their first renewal and 0.722 at each later one; and its surplus grows at 0.061 a year over the credit’s window, the return on funds included, a discount factor β of 0.941. The credit, whose means Table D.1 gives, falls toward zero over the final record years as the horizon closes.

³³Arnoud V. den Boer, “Dynamic Pricing and Learning: Historical Origins, Current Research, and New Directions,” *Surveys in Operations Research and Management Science* 20, no. 1 (2015): 1–18; Arnoud V. den Boer and Bert Zwart, “Simultaneously Learning and Optimizing Using Controlled Variance Pricing,” *Management Science* 60, no. 3 (2014): 770–783.

Table D.1: What the Kelly underwriter learned in the base arm, means over record years and seeds. A cell's factor multiplies the tracked expected loss (Equation D.6); its maximum is over every year and seed. The credit is per unit of net premium, one figure for all bands on new business (Equation D.3). Renewals form one cell, whose factor and credit stand for every renewal band. The deviation is the decision's grid choice of the logarithm of quoted over working premium, by band from smallest to largest, for renewals and new business, averaged over a year's fourth-quarter quotes before the floor and the filed band apply, the band bounding the deviation and schedule modification together to at most 0.223 in logs. Source: authors' calculations.

Quantity	Renewals, by size band			New business, by size band		
	Smallest	Middle	Largest	Smallest	Middle	Largest
Cell's factor, mean		1.430		1.385	1.255	1.111
Cell's factor, largest in any year		3.04		4.64	3.51	2.29
Renewal credit, per unit of premium net of expenses		0.523			0.280	
Deviation from the working premium, logarithm	0.221	0.205	0.134	0.200	0.212	0.216

The deviation chosen on the demand model stands near the band's limit of 0.223 on renewals in the two smaller bands and inside the band on the largest band's renewals; Table D.1 gives it by phase and band. The floor of Equation D.4 binds no quote of the base arm: with the credit in the floor, the deviated working premium stays above it, and the price of new business is set by the credit's divisor, the cells, and the deviation rather than by the floor. The premium the Kelly underwriter writes on new business is 1.30 times its own estimate of the expected loss, the same figure in each of the three size bands at the tables' precision; Table G.3 sets the two phases against the true expected loss on one basis.

The fitted moments of the estimator of §D.2.1 are of the kind the loss process has, and not all of its values: at the end of the record years it puts the persistence of the decaying part at 0.762 for frequency, where the process has 0.85, and at 0.356 for the log scale, below the process's value, and it finds a covariance between the two of -0.037 , the sign that a class defined on the sum of two independent traits induces within the class.

D.3 The Estimators

The estimator's observations are formed account-year by account-year. For an account-year s in a file let n_s be the reported claims with a positive value, q_s the expected reported claims at the class's frequency and the account's size, $\bar{y}_s$ the mean logarithm of the reported claim sizes, with $\sigma_c^2/2$ added for each open claim, and m_0 and v_0 the mean and variance of the logarithm of a claim on the bureau's curve at scale one before the occurrence limit, while the reported sizes are capped at it. The frequency observation is n_s/q_s , with noise variance $1/q_s$ around the year's relativity; the severity observation is $\bar{y}_s - m_0$ less the class's mean log scale, with noise variance $(n_s v_0 + n_s^2 \sigma_c^2)/n_s^2$, n_s^o the open claims among them. A year without a reported claim carries no severity observation. The class's mean log scale is the average of $\bar{y}_s - m_0$, weighted by n_s , over the account-years of the class in every block the Kelly underwriter has read, an account-year counting at each read.

Over every block of consecutive years the Kelly underwriter has read, it accumulates the products of the frequency observations less one at each lag k from 1 through 20, weighted by the expected claims, and the products of the centered severity observations at each lag, a year's observation weighted by $n_s/(n_s + 5)$, and it accumulates the product of the two kinds within an account.

To the frequency autocovariances it fits $A_f + B_f \rho_f^k$ by weighted least squares over a grid of ρ_f from 0.30 through 0.98 with A_f and B_f not below zero, and likewise $A_s + B_s \rho_s^k$ to the severity autocovariances. The cross moment is one constant, bounded by $\sqrt{A_f A_s}$ in absolute value. It refits after every quarter's reads.

For the predictor, with an account's file of w years let o be the vector of its $2w$ observations, centered, Σ their covariance under the fitted moments (the fitted autocovariances between years, the cross moment between kinds, and each observation's noise variance on the diagonal), and c_f and c_s the covariances of the observations with the coming year's frequency relativity and log scale. The tracked relativity is $1 + c'_f \Sigma^{-1} o$, not below 0.05, and the tracked log scale is the class's mean plus $c'_s \Sigma^{-1} o$, plus the covariance of the two that remains given the observations, with variance $A_s + B_s - c'_s \Sigma^{-1} c_s$. The expected limited claim and its second moment in Equation D.5 are integrals of the curve's closed forms over a normal distribution of the log scale with that mean and variance, by Gauss–Hermite quadrature.

In the demand model, a segment's reference is the logarithm of the working premium over the tracked estimate, whose part that changes between fits is the renewal cell's factor for renewals and, for new business, the band's cell factor over the credit's divisor. An observation recorded at one reference is moved, at each fit, by the difference between that reference and the present one, the on-level step of §D.2.3. For a segment, renewals or new business in a band, let quote j have log deviation ℓ_j from the working premium and weight ω_j . The log likelihood is

$$\sum_{j \text{ lost}} \omega_j \ln \phi\left(\frac{\zeta_j - \mu_\zeta}{\sigma}\right) - \sum_{j \text{ lost}} \omega_j \ln \sigma + \sum_{j \text{ bound}} \omega_j \ln \left[1 - \Phi\left(\frac{\ell_j - \mu_\zeta}{\sigma}\right)\right], \quad (\text{D.8})$$

with ζ_j the deviation ℓ_j less the broker's figure in logarithms and ϕ the standard normal density. With the log prior of §D.2.3 added, it is maximized in $(\mu_\zeta, \ln \sigma)$ by Newton's method, the covariance taken as §D.2.3 states. The pessimism variant of Section 5 lowers the conditional posterior mean of μ_ζ by one posterior standard deviation before the average of Equation D.7 is taken, which lowers the hold probability at every deviation.

D.4 The Variance Term

Without the variance term (Table D.2), three of the record's Kelly constructions, the Kelly underwriter as registered, the corrected Kelly underwriter on the growth discount, and learning at the Kelly decision with pessimism on the funds discount, choose another deviation on 0.018 to 0.029 of the quotes they compute over the record years, and for each the 90 percent confidence interval of the mean of its growth less its growth with the term lies inside $(-0.010, +0.010)$. By the reading rule, removing the term is shown not to move the mean growth of any of the three by more than 0.010. In the market of one tenth of the accounts the term is 0.6421 percent of the expected loss, 10.0 times its size in the record's market, and it changes the deviation on 0.201 of the Kelly underwriter's quotes; the 90 percent confidence interval of the mean of the Kelly underwriter's growth without the term less its growth with it, about a mean of +0.010, reaches beyond 0.010 and includes zero, and by the reading rule the effect is not shown.

Table D.2: The Kelly constructions without the variance term, each against the same construction with it: the share of the quotes it computes over the record years whose chosen deviation the term changes; and the fourth seat's growth without the term less its growth with it, paired by seed over 200 seeds, its mean, the 90 percent confidence interval of the mean, and its t statistic. The first three rows are in the record's market, against the arms of Section 5; the fourth is in the market of one tenth of the accounts. Source: authors' calculations.

Without the variance term, less the same construction with it	Quotes changed	Mean	90 percent interval	t
The Kelly underwriter as registered	0.018	+0.001	[−0.0038, +0.0054]	+0.30
The corrected Kelly underwriter, growth discount	0.029	−0.002	[−0.0069, +0.0032]	−0.61
Learning at the Kelly decision with pessimism, funds discount	0.028	0.000	[−0.0059, +0.0051]	−0.13
The Kelly underwriter as registered, one tenth of the accounts	0.201	+0.010	[−0.0084, +0.0288]	+0.91

E The Kelly Underwriter's Computation, Step by Step

The steps are in the order in which they are carried out. Each names the data it reads; all of it is data the insurer holds or that the market treats as public (§D.2).

At each quote, for each account:

1. Read the file. On an account the insurer holds: its own claim file for every year held, and the loss run received at acquisition for the five years before. On any other account: the broker's loss run of five years. Read the class and the bureau's claim frequency for it.
2. Track. Form the observations of §D.3 for each year of the file and compute the relativity, the log scale, and its variance from the fitted moments. Compute $\hat{x}_0$ and $\hat{v}$ by Equation D.5, and multiply both by the schedule modification.
3. Correct. Multiply both by the cell's factor: $\hat{x} = \kappa_c \hat{x}_0$, and $\hat{v}$ likewise, the cell being renewals for an account it holds and new business in the account's size band otherwise.
4. Set the working premium. $\hat{x}/(1 - \pi)$ on a renewal; on new business that amount divided by $1 + V_{nb}(1 - \varepsilon)/(1 - \pi)$, whatever the credit's sign.
5. Deviate. Take the deviation ℓ , on a grid over the filed band around the working premium, that maximizes $\bar{h}(\ell) G(P_\ell)$, with G of Equation D.2 and $\bar{h}$ the posterior-averaged hold probability of the account's segment (Equation D.7). Hold the deviation plus the logarithm of the schedule modification within $\ln(1 \pm 0.25)$.
6. Apply the floor. Quote the larger of the deviated working premium and the floor of Equation D.4, with S the insurer's surplus at the quote.

After each quarter's placements:

1. Add every file and loss run read in the quarter to the sums of the second moments, and refit them.
2. For the segment's demand model, record each quote that bound with its deviation, among them a renewal kept without going to market and one kept against a lower competing quote,

and each quote lost on an account that went to market with its deviation and the broker's figure, each brought on-level to the segment's reference; decay the weights by 0.98 a quarter and refit the posterior of each segment.

3. For each renewal quoted, record whether it was a first renewal and whether it was kept; for each kept, the premium net of expenses and Equation D.2's margin, first renewals and later ones apart.

At each year's close:

1. Update the cells by Equation D.6 on the policies of the past three policy years, each cell's reported losses against the expected reported losses on $\hat{x}_0$.
2. Update the renewal credit. Over the past three years, take p_1 and p_s as the shares of first and of later renewals kept, m_1 and m_s as the margins per unit of net premium on each, and $\hat{g}$ as the window's mean yearly log growth of surplus, the return on funds included; set $\beta = e^{-\hat{g}}$, the horizon H as the years to the final record year, and V_{ren} and V_{nb} by Equation D.3.

The fourth seat's two intermediate forms use parts of this list. The learned estimate at the plan's pricing carries out the first two steps of a quote and then prices as the Loss-ratio insurer does (Section 5). The Kelly decision on the plan's estimate replaces the first two steps by the plan's expected loss before any rate level, $L_c z M_{\text{exp}} M_{\text{sch}}$, and a variance from the class's claim frequency and the class's distribution of the severity trait, and carries out the rest. Learning at the Kelly decision (Section 5) computes the first form's price whole, by the same computation, and carries out step 5 around it with the anchored credit, and step 6 with the floor written with the quote-anchored credit (Section 4), on that form's expected loss and variance at its rate level, without the cells.

F The Registry of Predictions and Outcomes

Every prediction of the study was written down before the runs that tested it, under one rule: a predicted contrast holds where its paired mean over the 200 seeds has the predicted sign at $|t| > 2$, and a prediction that names another reading is read as it names. This appendix states the seven registrations in the order of their writing, each with its predictions, its outcomes under the rule, the order of writing against the order of running, and any reading of seeds outside the record that preceded it; the body reports each outcome once and points here. The record's names for the fillings of Section 5 are the learned-estimate seat for learning at the plan's pricing; the deviation policy for learning at the Kelly decision, corrected where it carries the anchored credit; and the standalone Kelly underwriter for the Kelly underwriter, corrected where it carries the anchored credit. The registration record, with the design note of each registration, travels with the paper's archive.

The first registration made 97 predictions for the two by two, the market around the Kelly underwriter, and the sensitivity program (Sections 5 and 7, Appendices G and H), registered after a disclosed reading of 20 seeds outside the record and before any arm ran on the record's seeds. Of them 80 held, 16 failed, among them the Kelly decision on the plan's estimate, the Kelly decision on the learned estimate, and the two by two's interaction, at the registered setting and at every other, and one, on the Bureau insurer's failures under a fixed level, is not tested, no failure having occurred in either arm.

The second registration made 10 for learning at the Kelly decision and the corrected Kelly underwriter on the funds discount, written after the first registration's outcomes were on the record. Eight were written after a disclosed reading of seeds outside the record: one, that its two arms of record reproduce their arms of the two by two run for run; six on four directions first read on those seeds, the correction on learning at the Kelly decision, its three leads over the traditional insurers, the pessimism variant on it, and learning at the Kelly decision with pessimism against the pricing it contains; and one, the funds discount on the Kelly underwriter, argued from the numeraire-portfolio property and read on those seeds before it was written. That reading ran six constructions of learning at the Kelly decision: the policy as first built, on each discount of the credit, and, on the funds discount, the policy without the credit, with the anchored credit, with the pessimism variant, and with both. The other two were written before the corrected Kelly underwriter had run anywhere: the correction and the pessimism variant on it. Of the 10, seven held and three failed: the funds discount of the credit, the pessimism variant on learning at the Kelly decision, and the primary prediction, learning at the Kelly decision with pessimism against the pricing it contains.

The third registration made two for learning at the Kelly decision on the growth discount, and the fourth four for the corrected Kelly underwriter on that discount, each written before any run of the arms it tests. Both of the third's failed: the discount's effect on learning at the Kelly decision and the primary prediction evaluated on that discount. Of the fourth's, three held, those on the correction on the registered Kelly underwriter, on the gap to learning at the plan's pricing, and on the discount's effect, and one failed, that on the pessimism variant.

The fifth registration made six for two of the four markets run beside the record, the market at a smaller surplus for its book and the market with a second seat that learns, written after a disclosed reading of 20 seeds outside the record and before the arms ran on the record's seeds; for its Kelly constructions without the variance term it states a reading rule in place of predictions (Section 6). All six held.

The sixth registration is an analysis on the records in hand, with no run of the market: the equivalence margin and the path metrics of §5.1, with six predictions written before any metric was computed. The primary contrast's value was on the record and is stated in the registration as known, so that the margin's reason stands on its own; the margin is 0.028 in the logarithm over the 28 record years, 2.8 percent of terminal surplus and a tenth of a percentage point of the mean geometric rate of growth a year, chosen in the objective's own unit, below the smallest effect any finding of the paper rests on, and below the correction's worth on the Kelly underwriter at either discount, the worth on the funds discount being one that a margin of a tenth of the learning's worth would have placed inside indifference. The first prediction is the equivalence reading itself at that margin; the second and third are read at $t > 2$, the paired within-path volatility and the Pitman-Morgan t of the dispersion across seeds; the fourth through sixth are read on medians and shares of the paired path, without a t . The equivalence was shown; of the five predictions of a direction, four held and one failed, that on the within-path volatility, which the Kelly route has below the plan route's where above was predicted.

The seventh registration ran the two routes at a second calibration of the filed band and the buyer's scale (Section 6), with six predictions written before its desk reading of 20 seeds outside the record, which is disclosed and left them unchanged, and before any arm ran on the record's seeds: the first, that the apparatus reproduces the record's arms at the record's setting unit for unit, read at the desk on 60 units and held there; the second, a loss for the Kelly route against the plan route at the wider band; the third and fourth, that the buyer's scale moves the Kelly route's mean renewal deviation and its band-edge share below their values at the record's scale at the

lower scale and above them at the higher; and the fifth and sixth, that the routes' contrast less the same contrast at the record's scale is below zero at the lower scale and above it at the higher. All six held on the record; at the desk reading one of them, the sixth, had failed, and the registration record states both readings. The registration also ran the Kelly route in markets E and C of the fifth registration, without a prediction.

The rule is applied to each prediction without adjustment for the number of predictions tested. At a Bonferroni level over the second registration's 10 contrasts of Table F.2, $|t| > 2.84$, no contrast of the ten moves across the threshold. At a Bonferroni level over each registration's own predictions of a direction, the rule's two-sided 5 percent divided among them, every prediction that held still holds: $|t| > 3.52$ over the 95 of the first registration, $|t| > 2.80$ over the nine of the second, its six predicted contrasts among the ten of Table F.2 and its three leads, $|t| > 2.52$ over the four of the fourth, $|t| > 2.66$ over the six of the fifth, $|t| > 2.26$ over the two of the sixth that are read by a t , and $|t| > 2.60$ over the five of the seventh; neither of the third's two held, and its level is $|t| > 2.26$. A prediction of a direction is every prediction except those read otherwise: the first registration's calibration and its prediction not tested, the second's and the seventh's predictions that arms reproduce arms, and the sixth's equivalence reading and its three predictions on medians and shares. A prediction that rests on three leads is one prediction of its family, and it holds at an adjusted level where each of its leads clears it. Over the 123 of the seven registrations together, at $|t| > 3.60$, every one that held still holds but the pessimism variant on the corrected Kelly underwriter on the funds discount ($t = +3.22$).

Table F.1: The two by two's contrasts and the Kelly underwriter's leads, the fourth seat's growth in one arm less its growth in another, paired by seed over 200 seeds: the mean on the logarithm (Equation A.3), its t statistic, and the seeds above zero. Learning alone is the learned estimate less the plan's, both at the plan's pricing; the Kelly decision on an estimate is the Kelly decision less the plan's pricing on that estimate, the decision carrying the quote-anchored credit; the contrast Both is the Kelly underwriter less a second Loss-ratio insurer; and the interaction is Both less the sum of learning alone and the Kelly decision on the plan's estimate. A lead is the Kelly underwriter's growth less a traditional insurer's in the same market. Contrasts are taken on unrounded values and may differ by 0.001 from differences of the means Figure 2 states. Source: authors' calculations.

Contrast	Mean	t	Seeds above zero
<i>The fourth seat's growth, one arm less another</i>			
Learning alone	+0.400	+31.10	197
The Kelly decision on the plan's estimate	-0.003	-0.24	108
The Kelly decision on the learned estimate	-0.244	-31.11	3
Both	+0.156	+11.25	153
Interaction	-0.241	-17.36	25
<i>The Kelly underwriter's growth less a traditional insurer's</i>			
Bureau insurer	+0.353	+22.73	190
Loss-ratio insurer	+0.430	+30.25	192
GLM insurer	+0.323	+22.35	186

Table F.2: The contrasts among the fillings of Table 1 that the second, third, and fourth registrations read, the fourth seat's growth paired by seed over 200 seeds: the mean on the logarithm (Equation A.3), its t statistic, and the number of seeds in which the contrast is above zero. The correction is an arm with the anchored credit less the same arm with the quote-anchored credit, the funds discount the Kelly underwriter on that discount less the registered one, the pessimism variant's contrast an arm with the variant less the same arm without it, and the growth discount on an arm the arm on the growth discount less the same arm on the funds discount. The second registration's six predictions are on the two corrections, the funds discount, the two contrasts of the pessimism variant, and learning at the Kelly decision with pessimism less learning at the plan's pricing; the final row of the first block is not among the ten contrasts the second registration read. Below the second subheading are the third and fourth registrations' contrasts, on the growth discount: the third's two predictions are on the growth discount on learning at the Kelly decision and on learning at the Kelly decision with pessimism less learning at the plan's pricing, the fourth's four on the correction on the Kelly underwriter as registered, the growth discount on the corrected Kelly underwriter, the pessimism variant on it, and the corrected Kelly underwriter less learning at the plan's pricing. Under the registered rule a predicted contrast holds where its paired mean has the predicted sign at $|t| > 2$; contrasts are taken on unrounded values. Source: authors' calculations.

Contrast, the fourth seat's growth	Mean	t	Seeds above zero
The correction, on learning at the Kelly decision	+0.110	+14.85	177
The correction, on the Kelly underwriter (funds discount)	+0.033	+7.09	134
The funds discount, on the Kelly underwriter	-0.013	-3.35	75
Pessimism, on learning at the Kelly decision	0.000	+0.03	100
Pessimism, on the corrected Kelly underwriter	+0.012	+3.22	119
<i>Less learning at the plan's pricing</i>			
Learning at the Kelly decision with pessimism	+0.011	+1.97	113
Learning at the Kelly decision	+0.011	+1.92	118
Learning at the Kelly decision, quote-anchored credit	-0.099	-12.01	34
The corrected Kelly underwriter	-0.224	-38.31	3
The corrected Kelly underwriter with pessimism	-0.212	-35.45	1
The Kelly underwriter, funds discount	-0.257	-36.35	1
<i>On the growth discount</i>			
The correction, on the Kelly underwriter as registered	+0.054	+11.03	164
The growth discount, on learning at the Kelly decision	-0.006	-1.36	86
The growth discount, on the same with pessimism	-0.005	-0.96	99
The growth discount, on the corrected Kelly underwriter	+0.034	+9.46	153
The growth discount, on the same with pessimism	+0.025	+6.43	136
Pessimism, on learning at the Kelly decision	+0.002	+0.42	98
Pessimism, on the corrected Kelly underwriter	+0.003	+0.80	110
<i>Less learning at the plan's pricing</i>			
Learning at the Kelly decision with pessimism	+0.006	+1.03	119
Learning at the Kelly decision	+0.004	+0.71	113
The corrected Kelly underwriter	-0.190	-30.88	3
The corrected Kelly underwriter with pessimism	-0.187	-31.30	2

Table F.3: The two routes' difference at the close of the record years and the equivalence test of the sixth registration: learning at the Kelly decision less learning at the plan's pricing, the fourth seat's growth paired by seed over 200 seeds, on each discount of the credit with and without the pessimism variant, and the Kelly underwriter as registered less learning at the plan's pricing. The mean is given on the logarithm (Equation A.3) and as the difference in terminal surplus in percent, the exponential of the mean less one; the standard error, the t statistic, and the 90 percent confidence interval are on the logarithm, the interval the mean plus and minus 1.645 standard errors. Equivalence is shown at the registered margin m where the interval lies inside $(-m, +m)$; the margin was registered with the primary pair's value on the record (Appendix F). Source: authors' calculations.

Less learning at the plan's pricing	Mean, logarithm	Percent of terminal surplus	s.e.	t	90 percent interval	At $m = 0.028$
<i>Learning at the Kelly decision</i>						
With pessimism, the funds discount	+0.0108	+1.1	0.0055	+1.97	[+0.0018, +0.0198]	shown
The funds discount	+0.0107	+1.1	0.0055	+1.92	[+0.0015, +0.0198]	shown
With pessimism, the growth discount	+0.0062	+0.6	0.0061	+1.03	[-0.0037, +0.0162]	shown
The growth discount	+0.0043	+0.4	0.0061	+0.71	[-0.0057, +0.0144]	shown
The Kelly underwriter as registered	-0.2440	-21.6	0.0078	-31.11	[-0.2569, -0.2311]	not shown

Table F.4: The fourth seat's two by two at five settings: the registered one, the Bureau insurer's refiling threshold at two other values, and the noise of a read of the account at two other values. Each cell is a contrast in the fourth seat's growth paired by seed over 200 seeds, the mean, with its t statistic in parentheses; the contrasts are those of Table F.1. Source: authors' calculations.

Setting	Learning	The Kelly decision on		Both	Interaction
	alone	the plan's estimate	the learned estimate		
Registered setting	+0.400 (+31.10)	-0.003 (-0.24)	-0.244 (-31.11)	+0.156 (+11.25)	-0.241 (-17.36)
Threshold 0.05	+0.394 (+31.42)	-0.008 (-0.75)	-0.241 (-32.74)	+0.153 (+10.94)	-0.233 (-17.39)
Threshold 0.15	+0.391 (+31.57)	+0.021 (+2.01)	-0.232 (-28.87)	+0.159 (+11.90)	-0.253 (-19.40)
Read noise 0.35	+0.490 (+36.08)	+0.044 (+3.58)	-0.278 (-31.83)	+0.212 (+13.67)	-0.322 (-22.16)
Read noise 1.05	+0.384 (+32.36)	-0.002 (-0.21)	-0.227 (-29.54)	+0.157 (+11.11)	-0.224 (-16.33)

G The Kelly Underwriter's Parts, and the Market around It

This appendix carries the arms and the tables behind Section 7: the Kelly underwriter with one part removed or one input changed (§G.1), and the market with the Kelly underwriter in the fourth seat against the market with a second Loss-ratio insurer there, on the price of new business (§G.2) and on turnover, shares, and rate levels (§G.3).

G.1 What Each Part Is Worth

The gap of Section 7 is not attributed among the Kelly underwriter's parts; the arms here remove one part at a time, the renewal credit, the demand model, and the cells' correction, then all three, and then change two inputs, the estimator's moments and the history every insurer reads (Section 7 summarizes). No direction was registered for these arms; Table G.1 reports the fourth seat's growth in each, its difference from the Kelly underwriter's paired by seed, and its leads.

Without the renewal credit the Kelly underwriter's growth falls by 0.224 ($t = -29.91$), to +0.467, and its three leads turn negative: -0.179 , -0.067 , and -0.172 (t of -10.42 , -3.94 , and -10.31). Without it the insurer binds 0.281 of its quotes as a challenger where the Kelly underwriter binds 0.679, and ends with 0.114 of the accounts. It is the one part whose absence turns the leads.

Table G.1: The Kelly underwriter with one part removed, all three removed, or one of two inputs changed. The columns give the fourth seat's growth; its growth less the Kelly underwriter's and its leads, paired by seed (t in parentheses); and its final share of accounts. Settled-sample moments freeze the estimator's class mean log scales and second moments at its own fit on a large settled sample; with five-year files every insurer reads five years of history, the traditional insurers on the plan as filed. Source: authors' calculations.

The fourth seat	Growth	Less the Kelly		Lead over (t)			Share of accounts
		underwriter (t)	Bureau	Loss-ratio	GLM		
Kelly underwriter	+0.691	—	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)		0.250
No renewal credit	+0.467	-0.224 (-29.91)	-0.179 (-10.42)	-0.067 (-3.94)	-0.172 (-10.31)		0.114
No demand model	+0.645	-0.046 (-5.72)	+0.481 (+46.45)	+0.507 (+49.52)	+0.485 (+46.68)		0.678
No cells' correction	+0.648	-0.042 (-3.67)	+0.539 (+40.10)	+0.548 (+43.27)	+0.547 (+44.21)		0.802
None of the three	+0.768	+0.077 (+7.84)	+0.587 (+59.86)	+0.617 (+68.34)	+0.590 (+61.87)		0.928
Settled-sample moments	+0.830	+0.139 (+15.43)	+0.541 (+55.36)	+0.616 (+74.67)	+0.498 (+54.73)		0.327
Five-year files for all	+1.029	+0.339 (+38.58)	+0.849 (+97.42)	+0.896 (+142.63)	+0.791 (+106.41)		0.414

Without the demand model the deviation is zero, and the insurer quotes its working premium. Its growth is lower by 0.046 ($t = -5.72$), and its three leads stay above zero with t of 46.45 or more. At the working premium it holds more accounts, 0.678 of them at the end, at a lower margin: its renewals are written at 1.51 times their true expected loss against 1.68 for the Kelly underwriter. The market's capital standard binds on its competitors: 25,435 new-business invitations are withheld by broker flight over the record years against 0 in the base arm.

Without the cells' correction the tracked estimate enters the price as it stands. The insurer's growth is lower by 0.042 ($t = -3.67$), and it prices below its competitors on a book whose losses run above its estimates: it ends with 0.802 of the accounts, the three traditional insurers hold 0.198 between them with growth of +0.109, +0.100, and +0.101, and broker flight withholds 75,696 of their new-business invitations as their capital ratios fall into the flight band. In no arm of this section does an insurer fail in any seed.

With the credit, the demand model, and the cells removed at once, what remains is the tracked estimate at a plan's provision with no rate level, no deviation, and no correction. That insurer grows by +0.768, which is +0.077 relative to the Kelly underwriter ($t = +7.84$). The removals are not additive: removing the three together raises growth where removing the credit alone lowers it, and in the construction the demand model's deviation and the cells' factors raise prices that the credit's divisor lowers. With that insurer in the fourth seat, accounts change insurer at 0.080 a year, the lowest rate of any arm of the record.

With the estimator's moments, each class's mean log scale and the second moments, frozen at its own fit to a sample of 40,000 accounts over 41 fully settled years, in place of the estimates it forms from the files it reads, the fourth seat grows by +0.139 more than the Kelly underwriter ($t = +15.43$): what estimating the moments on its own files costs the Kelly underwriter in this market, against the same estimator on a large sample; the frozen moments are the estimator's own in its form (§D.3), not the loss process's. With every insurer reading five years of an account's history and the traditional insurers rating by the experience plan as filed, the fourth seat grows by +0.339 more ($t = +38.58$), the largest movement of the section; the arm changes the reading of every insurer, not of the Kelly underwriter alone, so its leads are those of a different market, and the record carries the outcome and no mechanism for it.

G.2 The Price of New Business

Table G.2 gives the load each insurer charges on new business by size band, with its construction (Section 7 summarizes this appendix). The smallest band is the reference of the schedule credit and carries none (§A.3).

Table G.2: The load on new business in percent by insurer and size band (thirds of the year's accounts by manual premium), with the Kelly underwriter or a second Loss-ratio insurer in the fourth seat, each entry a mean over seeds of seed means over the record years. A traditional insurer's load is its new-business quote over its plan premium, less one: its schedule credit within the filed band, which bounds the credit and the schedule modification together, with the smallest band the credit's reference. The Kelly underwriter's is its new-business quote over its renewal price for the estimate, less one. Source: authors' calculations.

Insurer	With the Kelly underwriter			Without it		
	Smallest	Middle	Largest	Smallest	Middle	Largest
Bureau insurer	0.0	-7.5	-12.2	0.0	-1.2	-2.9
Loss-ratio insurer	0.0	-7.4	-12.1	0.0	-1.2	-2.8
GLM insurer	0.0	-11.0	-16.3	0.0	-2.7	-2.5
Fourth seat	-26.2	-31.5	-36.8	0.0	-1.3	-2.9

The Kelly underwriter's load is negative in every band and falls with size: -26.2, -31.5, and -36.8 percent in the smallest, the middle, and the largest band (t of -26.67, -87.18, and -197.19), and the smallest band's load exceeds the largest's by 10.6 percentage points ($t = +10.58$). Three learned quantities enter the level and the gradient: the renewal credit's divisor, one discount for all bands; the cells' factors, which are below the renewal cell's factor in every band of new business and higher in the smaller bands than in the largest (Table D.1), which lowers the price of new business against renewals and raises it on small accounts relative to large ones; and the deviations chosen on the demand model, which differ by phase and offset part of the cells' gradient (Table D.1). Against the true expected loss of the business written, the Kelly underwriter's price of new business is 1.23 in the smallest band and 1.29 in the largest, and its renewal price 1.63 and 1.70.

The traditional insurers' schedule credits also price new business below renewals, and by more where they compete with the Kelly underwriter: their load in the smallest band exceeds that in the largest by 13.5 percentage points on the mean of the three ($t = +97.98$), against 2.7 percentage points where the fourth seat is one of their own kind ($t = +29.91$), and the largest band's load is higher by 10.8 percentage points without the Kelly underwriter ($t = +65.80$). The schedule credit

follows the winning prices an insurer sees (§A.3); the record does not separate, in either market, the causes of the gradient among the competitors' information, the number of competing quotes, which rises with size in both markets, and the experience modification's credibility, which also rises with size.

Table G.3 sets the two phases of a policy side by side, on the true expected loss of the business written, which the simulation records and no insurer observes.

Table G.3: New business and renewals by insurer, sums over record years and seeds, with the Kelly underwriter or a second Loss-ratio insurer in the fourth seat. Price is premium over its true expected loss, the loss ratio is ultimate losses with loss adjustment expense over premium, and selection is the true expected loss over the writer's estimate. The share bound is, on new business, the quotes bound over the challenger quotes on accounts that went to market and were placed, and on renewals, the renewals kept over the renewal quotes. Source: authors' calculations.

Insurer	New business				Renewals			
	Price	Loss ratio	Selection	Bound	Price	Loss ratio	Selection	Bound
<i>With the Kelly underwriter</i>								
Bureau insurer	1.39	0.721	0.976	0.241	1.42	0.701	1.064	0.581
Loss-ratio insurer	1.39	0.721	0.974	0.199	1.43	0.701	1.061	0.553
GLM insurer	1.34	0.745	0.937	0.406	1.40	0.714	1.081	0.662
Fourth seat	1.28	0.781	1.012	0.679	1.68	0.595	1.017	0.592
<i>Without it</i>								
Bureau insurer	1.45	0.691	1.015	0.252	1.51	0.665	1.007	0.870
Loss-ratio insurer	1.46	0.684	1.011	0.218	1.51	0.662	1.002	0.847
GLM insurer	1.42	0.707	1.007	0.274	1.50	0.667	1.013	0.862
Fourth seat	1.46	0.687	1.012	0.231	1.51	0.662	1.003	0.853

The Kelly underwriter writes new business at 1.28 times its true expected loss and renewals at 1.68, a loss ratio of 0.781 on new business and 0.595 on renewals. In the market without it, the traditional insurers' loss ratios, across insurers and phases, lie within 0.045 of premium of one another. A new-business loss ratio above the renewal loss ratio, the pattern measured in insurers' books, holds for every insurer in both markets, by between 0.020 and 0.040 of premium for those that price by a plan and by 0.185 for the insurer that values retention. The Kelly underwriter's gap comes through the three learned quantities stated above, channels the record does not separate. In the market without a Kelly underwriter, every insurer's new-business selection, the true expected loss of the business it writes over its own estimate, is above one: the business an insurer wins runs above its own estimate, by 1.5 percent for the Bureau insurer. With the Kelly underwriter in the market the traditional insurers' selections on new business fall below one, and on renewals they rise (1.064 for the Bureau insurer, against 1.007): the new business the plans win is business their estimates overstate, and the renewals they keep are renewals their estimates understate; the record does not separate the mechanism.

G.3 Turnover, Shares, and Rate Levels

Table G.4 compares the two markets; Figure 4 draws the shares and the rate levels over the record years (Section 7).

Table G.4: The market with the Kelly underwriter or a second Loss-ratio insurer in the fourth seat, over 28 record years and 200 seeds. Rates per account and year are totals over seeds and years divided by the account-years; a challenger's quote is one by an insurer other than the holder on an account that went to market. Shares and rate levels are means at the end of the record years, and refilings the Bureau insurer's mean count a seed over the record years. The shares bound are in Table G.3, and growth in Figure 2 (its first and fourth panels). Source: authors' calculations.

Quantity	With the Kelly underwriter	Without it
<i>Per account and record year</i>		
Accounts changing insurer	0.394	0.141
Accounts going to market	0.491	0.266
Challengers' quotes	1.082	0.580
<i>Share of the accounts at the end of the record years</i>		
Bureau insurer	0.177	0.274
Loss-ratio insurer	0.161	0.222
GLM insurer	0.411	0.281
Fourth seat	0.250	0.223
<i>Rate level at the end of the record years</i>		
Bureau insurer	1.385	1.236
Loss-ratio insurer	1.378	1.259
GLM insurer	1.377	1.335
Bureau insurer's refilings a seed, record years	2.18	1.39

In the market without a Kelly underwriter, accounts change insurer at the calibrated rate. The buyer's costs were calibrated, on seeds outside the record, so that 0.14 of the accounts of the market of four plan-pricing insurers change insurer a year; on the record's 200 seeds 0.1410 of that market's accounts change insurer a year (standard error over seeds 0.0009), within two standard errors of the target, and the first registration's prediction held. With the Kelly underwriter in the fourth seat 0.394 of the accounts change insurer a year, and 0.491 go to market against 0.266: the turnover of the market with the Kelly underwriter is an outcome, not a calibration. The Kelly underwriter binds 0.679 of the quotes it makes as a challenger, and Table G.3 gives each insurer's share bound in the two markets.

The Kelly underwriter holds 0.250 of the accounts at the end of the record years (Table G.4). The GLM insurer holds the largest share in both markets, 0.411 with the Kelly underwriter and 0.281 without it, and the Kelly underwriter, on a smaller book than the GLM insurer's, writes its renewals at a higher price over their true expected loss than any traditional insurer (Table G.3). The traditional insurers' rate levels end higher than in the market without it (Table G.4). The book a traditional insurer keeps costs more against its plan, as the Bureau insurer's renewal selection shows (§G.2), and its method raises its level: the Bureau insurer refils 2.18 times a seed over the record years, against 1.39. The three insurers' growth is lower by 0.275, 0.246, and 0.217 than in the market without a Kelly underwriter (t of 22.69, 21.54, and 17.79), and it remains positive: each still earns more than the return on its surplus. In four of the 200 seeds the Bureau insurer holds under 1 percent of the accounts at the end of the record years. The capital standard does not bind in either market of this comparison: broker flight withholds no invitation, the IRIS stop closes no quarter, and no insurer fails; it binds mainly where an arm concentrates the accounts on one book (§8.2), and among the arms where it binds are those of learning at the plan's pricing, three of §G.1, one of the rules for the load (Table H.2), and the market with a Bureau insurer that holds a fixed level (§H.3).

The two markets differ from the first record year (Figure 4). The practice years, in which estimates, rate levels, and books carry forward while surplus is reset, have sorted the accounts before the record years begin: the Kelly underwriter holds 0.386 of them at the close of the first record year and between 0.250 and 0.422 throughout. Its share falls in each of the final nine record years, and each traditional insurer's rises in each of the final six; the record does not attribute the movement. The traditional insurers' rate levels stand higher from the first year, at 1.299, 1.360, and 1.346 against 1.225, 1.272, and 1.271, and they reach their highest in different years: the Bureau insurer's in the final record year, the GLM insurer's in year 22, and the Loss-ratio insurer's in year 16. The Loss-ratio insurer, whose complement of credibility reads competitors' rates, ends the record years with its level 0.018 above that at the close of the first year.

H The Sensitivity Program, Arm by Arm

Tables H.2–H.4 give arm by arm the sensitivity program that Table H.1 reports by group and Section 7 summarizes; §H.3 sets the market beside the two other first seats, and §H.4 states the sets of the record and its seeds. Appendix F states the registered predictions and their outcomes. The program's 79 arms are the 75 that Table H.1 groups, the two other first seats of §H.3, and the two inputs changed in Appendix G (Table G.1).

H.1 The Program by Group

Table H.1: The Kelly underwriter's lead over each traditional insurer across the 75 arms that change one condition, rule, or threshold, the read noise, or one fixed constant or choice, by group: the number of arms, the leads (three an arm) that are above zero at $t > 2$, and the lowest lead of the group by its t statistic, with its mean and the insurer it is over. Each arm runs the Kelly underwriter's market on seeds 0 through 199 with one assumption changed (Table A.1 states the one exception); a lead is paired by seed. Source: authors' calculations.

Group of arms	Arms	Leads above zero	Lowest lead	
		at $t > 2$	Mean (t)	Over
Conditions, the loads, the rules, the threshold	19	57 of 57	+0.091 (+5.80)	Bureau
Noise of a read of the account	2	6 of 6	+0.319 (+21.82)	GLM
Loss process, files, buyer's scale	18	54 of 54	+0.131 (+10.49)	GLM
Counts of years	15	45 of 45	+0.271 (+17.59)	Bureau
Claim sizes, report pattern, the return, the expense	9	27 of 27	+0.267 (+16.89)	Bureau
Other choices of Table A.1	12	36 of 36	+0.083 (+6.54)	GLM

The 19 arms of Table H.1's first group change the buyer's costs (recalibrated to annual switching of 0.11, and set to zero, at which 0.216 of the traditional market's accounts change insurer a year, short of the target of 0.25 the first registration named), the rate of remarketing unrelated to price, the rounding of the broker's figure, the claim rate, the spread of account size, how steeply invitations rise with size, a filed load on new business at 0.05 and at 0.1, the three other rules for the load, and the Bureau insurer's refiling threshold. In each, the three leads are above zero at $t > 2$ (57 of 57); the lowest, +0.091 ($t = +5.80$), is the lead over the Bureau insurer where account sizes are spread at 1.5 log standard deviations. A filed load raises the leads: the lowest lead is +0.441 at a filed

load of 0.05 and +0.604 at 0.1, against +0.323 with no load; under the rule that sets a traditional insurer's load from its own indication times the level of its competitors' winning prices, the three charge -22.6, -22.5, and -24.0 percent in the largest band. In the two noise arms the leads are above zero with a lowest t of 21.82, and one registered prediction failed: the Kelly underwriter's growth at a read noise of 1.05 less its growth at 0.35, predicted above zero, is -0.002 ($t = -0.27$), and the record does not distinguish the growth at the two levels.

The 54 arms that move one fixed constant or choice, the table's final four groups, carry one registered prediction each, that the arm's three leads are above zero at $t > 2$, and every one held: 162 of 162 leads above zero, the lowest t 6.54, on the lead over the GLM insurer where the schedule band is 0.4 either way. The Kelly underwriter's own growth moves most with the band's width, 0.217 lower at the wider band, over which its deviations range, and with the frequency trait: higher by 0.213 at a yearly shock of 0.45 and by 0.095 at a persistence of 0.925, lower by 0.142 and 0.128 at a shock of 0.15 and a persistence of 0.425: the larger the yearly change in an account's claim frequency and the longer the change persists, the more the Kelly underwriter grows; the severity trait's constants move its growth the other way, except at a yearly shock of 0.45, where the change, +0.005 ($t = +0.91$), has the frequency trait's sign and is not distinguished from zero (Table H.3). No other constant of the group moves the growth by more than 0.092.

H.2 The Arms

Table H.2: The arms that change one condition, one rule, one threshold, or one alternative of design: the Kelly underwriter's growth less its growth in the base arm, +0.691, paired by seed, and its lead over each traditional insurer paired by seed, means with t statistics in parentheses, over 200 seeds. An arm's label gives the value it moves an assumption to (Table A.1 gives the base value); §G.1 describes the arms of settled-sample moments and of five-year files. Source: authors' calculations.

Arm	Growth less the base arm's (t)	Lead over (t)		
		Bureau	Loss-ratio	GLM
The base arm	—	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)
Buyer's costs at annual switching of 0.11	+0.033 (+6.86)	+0.405 (+25.19)	+0.466 (+30.57)	+0.367 (+24.88)
Buyer's costs at zero, annual switching of 0.216	−0.046 (−9.80)	+0.309 (+22.26)	+0.379 (+28.10)	+0.268 (+21.52)
Remarketing unrelated to price 0.05	−0.020 (−4.79)	+0.349 (+22.31)	+0.415 (+27.63)	+0.316 (+22.19)
Remarketing unrelated to price 0.15	+0.026 (+5.99)	+0.397 (+26.72)	+0.450 (+32.75)	+0.324 (+24.16)
Broker's figure rounded to 0.1	+0.007 (+1.95)	+0.362 (+22.62)	+0.440 (+30.76)	+0.334 (+22.98)
Broker's figure exact	+0.005 (+1.50)	+0.366 (+24.35)	+0.428 (+30.14)	+0.327 (+23.85)
Claims a year at the median size 0.5	−0.028 (−3.06)	+0.378 (+24.02)	+0.372 (+24.50)	+0.339 (+21.46)
Claims a year at the median size 1.5	+0.008 (+1.00)	+0.331 (+24.66)	+0.435 (+37.92)	+0.323 (+26.62)
Account size, log spread 0.5	−0.038 (−3.87)	+0.309 (+23.53)	+0.351 (+31.32)	+0.301 (+26.81)
Account size, log spread 1.5	−0.086 (−11.18)	+0.091 (+5.80)	+0.196 (+13.27)	+0.351 (+22.03)
Invitations, power of the rank 0.5	+0.180 (+31.80)	+0.549 (+33.92)	+0.597 (+38.41)	+0.518 (+33.24)
Invitations, power of the rank 2	−0.147 (−31.12)	+0.217 (+16.32)	+0.273 (+20.62)	+0.229 (+18.21)
Filed load on new business 0.05	+0.130 (+24.50)	+0.505 (+34.08)	+0.563 (+40.56)	+0.441 (+34.11)
Filed load on new business 0.1	+0.287 (+42.55)	+0.691 (+50.10)	+0.723 (+53.73)	+0.604 (+48.40)
Load: the filed load, no credit	+0.402 (+65.00)	+0.726 (+51.02)	+0.796 (+63.18)	+0.824 (+63.49)
Load: the insurer's own indication	+0.159 (+18.54)	+0.544 (+39.84)	+0.582 (+41.48)	+0.642 (+49.69)
Load: indication and competitors' level	−0.185 (−28.70)	+0.241 (+21.24)	+0.254 (+24.16)	+0.286 (+26.08)
Refiling threshold 0.05	+0.021 (+5.55)	+0.417 (+27.72)	+0.430 (+30.52)	+0.329 (+22.93)
Refiling threshold 0.15	−0.036 (−8.25)	+0.256 (+15.49)	+0.426 (+30.12)	+0.333 (+23.33)
Noise of a read of the account 0.35	+0.004 (+0.66)	+0.405 (+24.76)	+0.462 (+31.51)	+0.319 (+21.82)
Noise of a read of the account 1.05	+0.002 (+0.42)	+0.343 (+23.44)	+0.411 (+28.00)	+0.328 (+23.60)
Bureau insurer at a fixed level	−0.261 (−45.26)	−0.009 (−0.57)	+0.312 (+31.75)	+0.307 (+26.97)
A Loss-ratio insurer in the first seat	+0.021 (+5.02)	+0.426 (+28.93)	+0.429 (+29.75)	+0.330 (+24.98)
Settled-sample moments	+0.139 (+15.43)	+0.541 (+55.36)	+0.616 (+74.67)	+0.498 (+54.73)
Five-year files for all	+0.339 (+38.58)	+0.849 (+97.42)	+0.896 (+142.63)	+0.791 (+106.41)

Table H.3: The arms that move one fixed constant of the loss process, the files, the buyer, the counts of years, the claim sizes, the report pattern, the return, or loss adjustment expense: the Kelly underwriter's growth less its growth in the base arm, +0.691, and its lead over each traditional insurer, both paired by seed, means with t statistics in parentheses, over 200 seeds. An arm's label gives the value it moves its constant to (Table A.1 gives the base value); the shares of claims reported by the ends of years one through three are 0.325, 0.775, 0.925 on the slower report pattern and 0.775, 0.925, 0.975 on the faster. The loss-run arms keep at five years an incumbent's run from acquisition, and the severity-trait arms the claim-size factor at its base value. Source: authors' calculations.

Arm	Growth less the base arm's (t)	Lead over (t)		
		Bureau	Loss-ratio	GLM
The base arm	—	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)
Frequency trait, spread 0.2	0.000 (−0.04)	+0.385 (+32.41)	+0.432 (+39.34)	+0.317 (+35.11)
Frequency trait, spread 0.6	0.000 (−0.02)	+0.341 (+17.33)	+0.412 (+23.78)	+0.320 (+17.52)
Frequency trait, persistence 0.425	−0.128 (−14.54)	+0.177 (+13.32)	+0.253 (+22.25)	+0.186 (+15.01)
Frequency trait, persistence 0.925	+0.095 (+10.21)	+0.492 (+28.66)	+0.556 (+34.90)	+0.510 (+33.00)
Frequency trait, yearly shock 0.15	−0.142 (−16.00)	+0.164 (+12.28)	+0.227 (+20.70)	+0.131 (+10.49)
Frequency trait, yearly shock 0.45	+0.213 (+23.73)	+0.695 (+47.74)	+0.737 (+58.83)	+0.653 (+47.19)
Severity trait, variance 0.2	+0.042 (+7.46)	+0.438 (+26.99)	+0.487 (+33.96)	+0.361 (+27.56)
Severity trait, variance 0.6	−0.024 (−4.81)	+0.332 (+21.25)	+0.395 (+29.28)	+0.299 (+23.11)
Severity trait, persistence 0.425	+0.068 (+11.91)	+0.446 (+27.80)	+0.516 (+37.46)	+0.405 (+29.08)
Severity trait, persistence 0.925	−0.043 (−6.58)	+0.357 (+29.05)	+0.402 (+35.65)	+0.306 (+26.16)
Severity trait, yearly shock 0.15	+0.051 (+9.79)	+0.420 (+28.34)	+0.493 (+34.03)	+0.373 (+26.93)
Severity trait, yearly shock 0.45	+0.005 (+0.91)	+0.409 (+30.90)	+0.454 (+38.12)	+0.365 (+30.39)
Case-reserve error, loss run 0.15	+0.012 (+2.77)	+0.371 (+23.33)	+0.439 (+29.94)	+0.332 (+22.57)
Case-reserve error, loss run 0.45	+0.001 (+0.29)	+0.366 (+23.56)	+0.421 (+29.50)	+0.321 (+22.92)
Case-reserve error, own file 0.075	+0.007 (+1.74)	+0.368 (+22.81)	+0.428 (+30.23)	+0.331 (+24.28)
Case-reserve error, own file 0.225	+0.002 (+0.43)	+0.360 (+22.71)	+0.432 (+32.22)	+0.324 (+24.37)
Buyer's response scale 0.025	−0.021 (−5.10)	+0.348 (+22.06)	+0.422 (+28.90)	+0.291 (+20.69)
Buyer's response scale 0.075	+0.027 (+5.79)	+0.375 (+25.24)	+0.442 (+31.06)	+0.372 (+29.25)
Bureau's statement, policy years 3	−0.007 (−1.64)	+0.347 (+21.78)	+0.424 (+29.38)	+0.325 (+23.33)
Bureau's statement, policy years 7	+0.006 (+1.43)	+0.366 (+24.23)	+0.438 (+31.95)	+0.317 (+24.17)
Bureau's statement, lag in years 3	+0.006 (+1.46)	+0.369 (+22.74)	+0.439 (+31.30)	+0.328 (+23.23)
Loss-ratio method's window, years 3	−0.017 (−3.99)	+0.350 (+23.60)	+0.415 (+30.77)	+0.309 (+22.03)
Loss-ratio method's window, years 7	+0.008 (+2.12)	+0.387 (+24.98)	+0.395 (+26.24)	+0.336 (+24.28)
Cells' window, years 2	−0.028 (−6.76)	+0.320 (+20.37)	+0.398 (+26.80)	+0.286 (+19.04)
Cells' window, years 4	+0.018 (+4.98)	+0.381 (+24.96)	+0.450 (+32.13)	+0.349 (+25.17)
Credit's window, years 2	+0.001 (+0.39)	+0.369 (+24.49)	+0.428 (+29.78)	+0.330 (+23.08)
Credit's window, years 4	+0.004 (+1.21)	+0.360 (+23.34)	+0.432 (+30.29)	+0.322 (+22.06)
Schedule fit's window, years 2	+0.006 (+1.52)	+0.360 (+24.27)	+0.438 (+30.94)	+0.331 (+22.82)
Schedule fit's window, years 4	−0.002 (−0.75)	+0.356 (+23.27)	+0.429 (+31.56)	+0.316 (+23.12)
Bureau insurer's experience, years 3	+0.022 (+4.88)	+0.484 (+30.33)	+0.409 (+27.85)	+0.304 (+22.50)
Bureau insurer's experience, years 7	−0.021 (−5.24)	+0.271 (+17.59)	+0.444 (+31.67)	+0.344 (+24.02)
Loss run, years 3	−0.085 (−12.87)	+0.272 (+20.47)	+0.338 (+26.98)	+0.227 (+17.74)
Loss run, years 7	+0.054 (+9.30)	+0.429 (+32.47)	+0.482 (+38.28)	+0.386 (+29.81)
Claim sizes, times 0.5	+0.020 (+3.25)	+0.465 (+33.15)	+0.489 (+37.47)	+0.398 (+32.66)
Claim sizes, times 1.5	−0.016 (−3.34)	+0.287 (+17.34)	+0.386 (+24.82)	+0.285 (+19.29)
Claim-size curve unscaled	+0.018 (+3.21)	+0.455 (+32.87)	+0.475 (+36.94)	+0.395 (+32.17)
Report pattern, slower	−0.062 (−11.66)	+0.267 (+16.89)	+0.335 (+23.12)	+0.241 (+17.41)
Report pattern, faster	+0.062 (+10.75)	+0.446 (+31.03)	+0.532 (+42.39)	+0.416 (+34.60)
Return on funds 0.0258	−0.064 (−14.66)	+0.329 (+20.66)	+0.399 (+27.21)	+0.330 (+21.72)
Return on funds 0.0365	+0.011 (+3.32)	+0.368 (+23.48)	+0.432 (+29.51)	+0.334 (+24.72)
Loss adjustment expense over indemnity 0.337090	+0.007 (+1.80)	+0.372 (+24.64)	+0.431 (+30.58)	+0.331 (+23.67)
Loss adjustment expense over indemnity 0.590157	−0.007 (−1.65)	+0.337 (+21.53)	+0.415 (+30.11)	+0.308 (+21.54)

Table H.4: The arms that move one of the other choices of Table A.1, among them three that move or remove a rule of the capital standard: the Kelly underwriter's growth less its growth in the base arm, +0.691, paired by seed, and its lead over each traditional insurer paired by seed, means with t statistics in parentheses, over 200 seeds. An arm whose mechanism the base arm never reaches (broker flight removed, the IRIS stop removed) reproduces the base arm's record seed for seed, and its contrast has no t . Source: authors' calculations.

Arm	Growth less the base arm's (t)	Lead over (t)		
		Bureau	Loss-ratio	GLM
The base arm	—	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)
Schedule band, either way 0.4	−0.217 (−37.81)	+0.132 (+10.03)	+0.179 (+13.76)	+0.083 (+6.54)
Experience plan's K (\$) 100,000	+0.003 (+0.66)	+0.368 (+24.43)	+0.427 (+28.19)	+0.326 (+24.47)
Profit provision 0.02	−0.081 (−20.42)	+0.312 (+21.02)	+0.380 (+27.67)	+0.311 (+22.55)
Profit provision 0.06	+0.092 (+23.65)	+0.426 (+26.71)	+0.483 (+34.23)	+0.351 (+27.01)
Expenses, share of premium 0.258	−0.008 (−2.00)	+0.357 (+22.73)	+0.420 (+29.61)	+0.315 (+23.08)
Broker flight, band shifted upward	0.000 (+0.10)	+0.353 (+22.79)	+0.431 (+30.20)	+0.321 (+22.01)
Broker flight removed	identical	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)
The IRIS stop removed	identical	+0.353 (+22.73)	+0.430 (+30.25)	+0.323 (+22.35)
Demand model's prior spreads times 0.5	+0.005 (+1.10)	+0.369 (+24.42)	+0.431 (+30.19)	+0.332 (+25.11)
Demand model's prior spreads times 1.5	+0.001 (+0.16)	+0.356 (+23.12)	+0.429 (+29.97)	+0.320 (+20.80)
Demand model's decay, half-life halved	+0.001 (+0.38)	+0.363 (+23.84)	+0.424 (+29.71)	+0.317 (+22.13)
Demand model's decay, half-life doubled	+0.007 (+2.54)	+0.366 (+24.02)	+0.435 (+31.14)	+0.334 (+23.69)

H.3 Two Other First Seats

The first seat holds the Bureau insurer of §A.3, which refiles its multiplier; two other first seats are on the record. Where the Bureau insurer holds a level of one throughout, it holds 0.687 of the accounts at the end of the record years. The Loss-ratio insurer's and the GLM insurer's levels end at 1.406 and 1.752, against 1.378 and 1.377 in the base arm; broker flight withholds 39,555 new-business invitations over the record years; and no insurer fails. The Kelly underwriter's growth falls to +0.430, against +0.691 beside a refiling Bureau insurer (the base arm's growth less this arm's, paired by seed, +0.261 at $t = +45.26$; Table H.2 prints the reverse difference). Its lead over the fixed-level insurer, −0.009 ($t = -0.57$), is the one lead of the sensitivity program not above zero, while its leads over the other two stand (+0.312 and +0.307). Where the first seat is a second Loss-ratio insurer, so that no insurer holds a fixed level, the three leads are +0.426, +0.429, and +0.330, each above zero.

H.4 The Sets of the Record

The record is 114 arms of 200 seeds each, 22,800 runs of the market, in nine sets: 34 arms that hold the two by two, the Kelly underwriter's parts and alternatives, the conditions, the loads, the rules (the three other rules for the load each also in the market without the Kelly underwriter, arms no table reports), the thresholds, and the two other first seats; 14 that hold the two by two's three other arms at the two other thresholds and all four of its arms at two other levels of read noise; 18, 15, and nine that move the fixed constants of the loss process, the files, the buyer, the counts of years, the claim-size curve, the report pattern, the return, and the loss adjustment expense; 12 that move the other choices of Table A.1; the second registration's eight that hold learning at the Kelly decision and the corrected Kelly underwriter on the funds discount, with their controls (Section 5); two that run learning at the Kelly decision on the growth discount; and two that run the corrected

Kelly underwriter on that discount (Section 4). Beside the record, the fifth registration's 21 arms of 200 seeds, 4,200 runs, are the four markets and the Kelly constructions without the variance term of §D.4, and the seventh registration's 11 arms of 200 seeds, 2,200 runs, are the two routes at a second calibration and the Kelly route in markets E and C (Section 6). Every run is a function of its arm and its seed alone, in double precision on one thread.

I The Quote Data and the Further Markets

Table I.1 gives the quotes of learning at the Kelly decision by arm and phase, whose renewal shares at the band's upper end and at the zero deviation Figure 3 draws (§5.2); Table I.2 gives the Kelly route's quote data at each setting of the seventh registration (Section 6); and Table I.3 gives the fourth seat's contrasts and the Kelly underwriter's leads in the four further markets of the fifth registration (Section 6), which the paragraph before it reads.

Table I.1: The quotes of learning at the Kelly decision over the record years and the 200 seeds, by arm (above the subheading, on the funds discount, the policy with the quote-anchored credit, with the anchored credit, and with the anchored credit and the pessimism variant, the arms of Table 1) and phase: the mean deviation from the plan route's price (a logarithm); the share of quotes within one grid step (0.0025 in log price) of the filed band's upper end, the share at exactly the zero deviation, and the share raised to the floor; for the corrected arms, the posterior-mean hold probability at the chosen deviation (predicted hold); and the share of the renewal quotes that held the account (realized hold). On new business the predicted hold is over every account the policy priced and the record's share held is over the quotes it made, so no realized hold is set beside it. Below the subheading, the corrected arms of the third registration, on the growth discount. Source: authors' calculations.

Arm	Phase	Mean deviation	At the band's upper end	At zero	Raised to the floor	Predicted hold	Realized hold
Quote-anchored	Renewals	+0.174	0.288	0.127	0.000	—	0.753
	New business	+0.021	0.105	0.169	0.000	—	—
Corrected	Renewals	+0.120	0.278	0.166	0.000	0.775	0.809
	New business	−0.081	0.039	0.136	0.000	0.708	—
Corrected, with pessimism	Renewals	+0.118	0.276	0.168	0.000	0.776	0.811
	New business	−0.088	0.037	0.133	0.000	0.714	—
<i>On the growth discount</i>							
Corrected	Renewals	+0.135	0.288	0.150	0.000	0.760	0.793
	New business	−0.057	0.039	0.146	0.000	0.680	—
Corrected, with pessimism	Renewals	+0.133	0.288	0.152	0.000	0.761	0.795
	New business	−0.063	0.037	0.145	0.000	0.686	—

Table I.2: The quote data of learning at the Kelly decision with pessimism at each setting of the seventh registration, on the basis of the readout of record, each seed's mean over its record years and the mean of those over the 200 seeds (Table I.1 pools the quotes): the mean renewal deviation from the plan route's price (a logarithm), the share of renewal quotes within one grid step (0.0025 in log price) of the filed band's upper end, and the mean new-business deviation; and, for the mean renewal deviation and the band-edge share, the movement from the record's setting paired by seed, with its t statistic. Source: authors' calculations.

Setting	Mean renewal	Share at the	Mean new-business	Movement from the record's setting (t)	
	deviation	band's upper end	deviation	Renewal deviation	Band-edge share
The record's setting	+0.119	0.275	-0.093	—	—
The band of 40 percent either way	+0.117	0.125	-0.023	-0.0013 (-3.48)	-0.1500 (-123.24)
The buyer's scale of 0.025	+0.113	0.254	-0.036	-0.0059 (-21.33)	-0.0209 (-15.74)
The buyer's scale of 0.075	+0.123	0.285	-0.113	+0.0041 (+15.56)	+0.0109 (+11.95)

In each of the four markets learning at the plan's pricing outgrows a second Loss-ratio insurer in the fourth seat, and the Kelly underwriter's growth is below it, each at $|t| > 2$ (Table I.3); at the smaller surplus and beside a second seat that learns both were registered predictions. At the smaller surplus, as registered, and in the market of one tenth of the accounts the Kelly underwriter leads the three traditional insurers. Beside a second seat that learns it leads the Bureau insurer and the Loss-ratio insurer and trails the second seat that learns (-0.221, $t = -11.94$). In that market's arm whose fourth seat is a second Loss-ratio insurer, the second seat that learns leads the Bureau insurer, the Loss-ratio insurer, and the fourth seat, as registered (+0.505, +0.624, and +0.612; t of +35.30, +44.34, and +48.19); in its arm whose fourth seat is learning at the plan's pricing, the two seats that learn are not distinguished in growth (the fourth seat less the second, -0.019, $t = -0.79$). Beside a second seat that learns, learning at the Kelly decision with pessimism on the funds discount is within the registered margin of learning at the plan's pricing (+0.003, $t = +0.44$), and under the severity trend it outgrows learning at the plan's pricing beyond the margin (+0.069, $t = +4.89$); in markets E and C the seventh registration ran it beside the plan route, and it leads there (Section 6). The other contrasts of the four markets were not predicted.

Table I.3: The four markets of the fifth registration: E, an endowment at which a seat writing the opening book stands at the capital ratio at which broker flight begins; C, one tenth of the accounts; L, the GLM insurer's seat a second seat that learns; T, a trend of 6.8 percent a year in the mean claim limited per occurrence. The fourth seat's contrasts within each market, learning at the plan's pricing in every row being the fourth seat so filled, and the Kelly underwriter's leads, the fourth seat's growth (Equation A.3) paired by seed over 200 seeds, each mean on the logarithm above its t statistic in parentheses; learning at the Kelly decision is the arm with pessimism on the funds discount, run in L and T by the fifth registration and in E and C by the seventh. The variance term at each market's mean endowment is in percent of the expected loss of an account at the curve's own scale of claim sizes. Source: authors' calculations.

The fourth seat's growth	E	C	L	T
Learning at the plan's pricing less a second Loss-ratio insurer	+0.451 (+26.37)	+0.241 (+12.01)	+0.340 (+23.11)	+0.163 (+7.39)
Kelly underwriter less learning at the plan's pricing	-0.245 (-22.28)	-0.145 (-8.26)	-0.212 (-21.98)	-0.510 (-40.77)
Learning at the Kelly decision less learning at the plan's pricing	+0.023 (+3.19)	+0.116 (+8.50)	+0.003 (+0.44)	+0.069 (+4.89)
<i>The Kelly underwriter's lead over</i>				
the Bureau insurer	+0.478 (+21.40)	+0.397 (+17.65)	+0.185 (+12.18)	-0.113 (-4.51)
the Loss-ratio insurer	+0.586 (+27.38)	+0.328 (+14.49)	+0.266 (+19.81)	-0.223 (-9.50)
the GLM insurer (in L, the second seat that learns)	+0.448 (+22.45)	+0.505 (+23.93)	-0.221 (-11.94)	-0.119 (-5.02)
Variance term, percent of expected loss	0.1372	0.6421	0.0641	0.0644

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